Chapter 10 – The End of the Radiation Era: Almost a Double Coincidence?

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1. The Time When The Radiation and Matter Mass-Energy Densities Are Equal

What do we mean by the end of the radiation era? The radiation is, after all, still with us. There is the same number of cosmic background photons now as when positron annihilation was completed a few minutes after the Big Bang. On the other hand, the photon temperature is now far lower. Hence, the total energy associated with the photons is now far lower. In contrast, the total energy of the baryons – including their rest mass energy - has changed little since the first second. The thermal energy (average $2.7k_BT$ for photons, $3.15k_BT$ for baryons) at one second was ~2.38MeV[1] for photons (2.77MeV for baryons). But this is small compared with the proton rest mass (~938.27MeV). Consequently the total energy (rest mass plus kinetic) of the baryons has changed little.

The vast numerical preponderance of the photons over the baryons (by a factor of 1.9×10^9) meant that the energy of the baryons at 1 second (938.27 + 2.77 = 942.0 MeV per baryon) was negligible compared to that of the photons ($1.9 \times 10^9 \times 2.38 = 4.5 \times 10^9$ MeV per baryon, i.e. 4.8×10^6 times greater than the baryon energy). This is in contrast to the situation today, for which the average CMB photon energy is only 0.24×10^{-3} eV, so the total photon energy per baryon is $1.9 \times 10^9 \times 0.24 \times 10^{-3} = 0.45$ MeV, only a fraction $\sim 0.5 \times 10^{-3}$ of the baryon rest mass energy. We therefore expect that at some intermediate time the total photon and baryon energies must have been equal.

The temperature, and hence the energy per photon, decreases in proportion to $1/\sqrt{t}$ whilst radiation is dominant. Hence, since the total baryon energy is roughly constant, and the photon energy starts, at 1 second, a factor 4.8×10^6 times greater, the total photon energy will equal the total baryon energy when $4.8 \times 10^6 / \sqrt{t} = 1$, i.e. at about 2.3×10^{13} seconds (~730,000 years, temperature ~ 2,130°K, average photon energy, $2.7k_BT$, ~0.5eV). Strictly, the photon temperature falls more quickly, proportional to $1/t^{2/3}$, when the temperature has dropped below 3000°K (see Chapters 3, 4). However, this makes no difference to the temperature at which the photon and baryon energies become equal (since this is just the temperature at 1 sec, i.e. $\sim 1.02 \times 10^{10}$ °K, divided by 4.8×10^6). But the time at which this temperature is reached is 615,000 years.

This is not quite right if we want the end of the radiation era. The neutrinos also count as radiation – or at least, they would if they had zero rest mass. The electron, muon and tau neutrinos appear to all be less massive than the ~0.5eV thermal energy at this time[2], and so they might reasonably still be regarded as 'radiation'.

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[1] Or rather, this is what it would have been if electron-positron annihilation had been complete at 1 second. Of course it was not (see Chapter 7). In truth, the average photon energy at 1 second would have been a factor $(4/11)^{1/3} = 0.714$ less, i.e. ~1.7MeV. However this forms a convenient fiction for scaling purposes. Exactly the same results would follow if we replaced this statement with "the average photon energy at (say) 1000 seconds was 0.075MeV".

[2] The sum of the three neutrino masses is currently believed to be <0.6eV (see Chapter 5)

If any of the neutrinos had been significantly more massive (say ~10eV, as required for neutrinos to account for the bulk of 'dark matter', see Chapter 5), then it would be more reasonable to count them on the 'matter' side of the equation rather than as radiation. This is dealt with below. For the present we note that if all 3 species of neutrino are much less massive than ~0.5eV, and hence are to be regarded as radiation, they will increase the total radiation energy by a factor of 1.681 (see Chapter 5). In this case, the time of matter / radiation energy-density equality increases to 6.5 x 10^{13} seconds (~2 Myrs, temperature ~1,265^{o}K ~ 990^{o}C, average photon energy, 2.7k_BT, ~0.3eV), assuming the radiation era dependence $T \propto 1/t^{1/2}$. Again, accounting for the reduction of temperature according to $T \propto 1/t^{2/3}$ below 3000^{o}K, a better estimate of the time at which this equality occurs is ~1.34 Myrs, but the temperature and thermal energies at equality are unchanged.

The above considerations fall short of truly equating $\rho_{radiation}$ and ρ_{matter}. Actually they equate $\rho_{radiation}$ and $\rho_{baryons}$. The latter accounts for essentially all of 'ordinary' matter. The only thing not explicitly accounted for are the electrons, but they make a correction of only ~0.05% because of their small mass compared with the nucleons. However, astronomers now believe that the bulk of matter is 'dark' and, moreover, that the bulk of this dark matter is non-baryonic. From WMAP/COBE the total matter density as a fraction of the critical density is Ω_M ~ 0.26 +/- 0.04, whereas Ω_b alone is about 0.04 (leaving Ω_{dark} ~ 0.22). Thus, there appears to be between 5.4 and 7.4 times more matter than accounted for by baryons alone. Taking account of the dark matter, the time at which $\rho_{radiation} = \rho_{matter}$ becomes, if neutrino radiation is ignored:-

(a) For 5.4 times more matter than baryonic matter: 7.9 x 10^{11} seconds (~25,000 years, temperature ~11,500^{o}K, 2.7k_BT, ~2.7eV);

(b) For 7.4 times more matter than baryonic matter: 4.2 x 10^{11} seconds (~13,340 years, temperature ~15,740^{o}K, 2.7k_BT, ~3.7eV).

It is more reasonable to include the three neutrino species as contributing to the radiation (making the total radiation energy 1.681 times the photon energy). The time at which $\rho_{radiation} = \rho_{matter}$ then becomes:-

(a) For 5.4 times more matter than baryonic matter: 2.2 x 10^{12} seconds (70,000 years, temperature ~6,830^{o}K, 2.7k_BT, ~1.6eV);

(b) For 7.4 times more matter than baryonic matter: 1.2 x 10^{12} sec (37,700 years, temperature ~9,360^{o}K, 2.7k_BT, ~2.2eV).

Hence, we have 'equality' times ranging from ~13,340 years up to ~1.34 million years, and corresponding temperatures from ~1,265^{o}K to ~15,740^{o}K. Which of these is the 'right' answer? Clearly, it depends upon the question! If we truly want to know the time/temperature when matter and radiation have the same mass-energy density, then 37,700 years at temperature 9,360^{o}K may be the best answer. This includes all matter and all radiation.

On the other hand, we may be interested in the influence of radiation on ordinary baryonic matter (e.g. as regards radiation pressure). Since the neutrinos do not interact significantly, they can be ignored if this is the issue. In this case the 'right' answer is 615,000 years and 2,130°K.

Finally, we may be interested in the formation of stars and galaxies, in which case the radiation pressure contribution of the neutrinos is negligible, but the contribution of dark matter (be it neutrinos or anything else) to gravitation is important. In this case the 'right' answer is 13,340 years and 15,740°K. This is, of course, far too early for stars to form.

The four permutations are summarised below:-

Radiation	Matter	Time of Equality	Temperature at Equality
γ	b	615,000	2,130
$\gamma + 3\nu$	b	1,340,000	1,265
γ	b + dark	13,340	15,740
$\gamma + 3\nu$	b + dark	37,700	9,360

These may be compared with the times/temperatures at which hydrogen recombines, i.e. 185,000 to 363,000 years and 3100 to 4300°K. Certainly the time of hydrogen recombination is within the range of times for which the matter and radiation mass-energy densities equate, but the spread of the latter means that this 'coincidence' is less than striking. In fact, the range of the recombination time does not coincide with any of the above four possibilities for radiation/matter equality. It would only do so if part, but not all, of the dark matter was counted.

2. The Origin of $t_{equal} \sim t_{recombination}$?

In the notation of Chapter 9, the condition that the radiation and matter mass-energy densities be the same can be written,

$$\mu \frac{\rho_\gamma}{\xi_{\gamma N}} M_p c^2 = 2.7kT\zeta\rho_\gamma \qquad (1)$$

where,

$\xi_{\gamma N}$ = the photon:baryon ratio;
μ = the ratio of all matter being counted to baryonic matter
ζ = a factor accounting for the neutrino contribution to radiation;
M_p = the mass of the proton

Hence, μ is 1 or 7.4, according to whether dark matter is excluded or included; ζ = 1, 1.454 or 1.681 according to whether no neutrinos, 2 neutrino species, or 3 neutrino species are included as radiation. The photon number density, ρ_γ, can clearly be cancelled from Equ.1. Re-arranging (1) to give the photon:baryon ratio in terms of the temperature T_{equal} at radiation/matter equality,

$$\xi_{\gamma N} = \left(\frac{\mu}{2.7\zeta}\right)\left(\frac{M_p c^2}{kT_{equal}}\right) = \left(\frac{\mu}{2.7\zeta}\right)\left(\frac{M_p}{m}\right)\lambda_{equal} \qquad (2)$$

where, $\lambda = mc^2 / kT$, and m is the electron mass.

Now a cheap way of gauging the recombination time is to compare the ionisation energy with the average energy of the photons. Thus, if photons, electrons and protons were comparably abundant, the recombination temperature would be given roughly by,

$$2.7kT_{recomb} = \chi = \alpha^2 mc^2 / 2 \qquad (3a)$$

However, we have seen that the huge numerical abundance of photons compared with electrons/ protons leads to much lower recombination temperatures. Only a very small proportion of the photons need have energies greater than the ionisation potential in order for the atomic hydrogen to re-ionise. In other words, the average photons energy will be quite a bit less than the ionisation potential. Hence we re-write (3a) as,

$$2.7kT_{recomb} = \kappa\chi = \kappa\alpha^2 mc^2 / 2, \quad \text{or} \quad \lambda_{recomb} = \frac{5.4}{\kappa\alpha^2} \qquad (3b)$$

Taking the correct recombination temperature to be 3,700^{o}K (when ~50% of protons have recombined), we find that $\kappa = 0.063$.

If we now make the hypothesis that the matter/radiation equality time is the same as the recombination time, then substituting (3b) into (2) gives,

$$\xi = \left(\frac{2\mu}{\zeta\kappa}\right)\frac{1}{\alpha^2}\left(\frac{M_p}{m}\right) \qquad (4)$$

Hence, if we include only photons as 'radiation' and only baryons as 'matter', i.e., $\mu = \zeta = 1$, then Equ.4 predicts the photon:baryon ratio to be 1.1 x 10^9, quite close to the observed value (1.9 x 10^9). However, κ is something of a fiddle factor – and the value adopted for it has actually assumed the observed photon:baryon ratio, so this agreement is not quite as impressive as it seems.

If we include all dark matter, and include three neutrino species as radiation, (i.e., $\mu = 7.4$, $\zeta = 1.681$) then Equ.4 predicts the photon:baryon ratio to be 4.8 x 10^9. Hence, for the range of possibilities for μ and ζ , the photon:baryon ratio is within a factor of 2 or so either side of the observed value.

Despite having introduced the fiddle factor κ, we note that κ is not terribly sensitive. One would guess that it would be of the order of ~0.1. Hence, in a sense, Equ.4 does betray the origin of the size of the photon:baryon ratio – at least if the identity of the radiation/matter equality time and the recombination time can be assumed. If an interesting universe requires this condition, then the strengths of the strong nuclear and electromagnetic interactions are tuned to give rise to a value for $\alpha^2 M_p / m$ about in line with its actual value (3.4 x 10^7).

Note that Equ.(4) is essentially the same as Davies's Equ.(4.25) ("The Accidental Universe").

3. Method of Section 2 Without The Fiddle-Factor

We may carry out the calculation of the implications of the hypothesis $t_{equal} = t_{recombination}$ without introducing the fiddle factor, κ. The value of λ at recombination may be found in terms of the photon:baryon ratio $\xi_{\gamma N}$ by using a simple approach which we have already seen to be quite accurate in Chapter 8. This involves equating the number of photons with energy greater than E_1 to the number of protons, where E_1 is the energy spacing between the hydrogen n = 1 and n = 2 levels. Thus,

Equ.(5)

$$0.416(2+2x_1+x_1^2)e^{-x_1}\rho_\gamma^N = \rho_p^N = f_p\rho_b^N = 0.875\rho_b^N \quad \text{hence} \quad 0.475(2+2x_1+x_1^2)e^{-x_1} = 1/\xi_{\gamma N}$$

where,
$$x_1 = \frac{3}{4}\frac{\chi}{kT_{recomb}} = \frac{3}{8}\frac{\alpha^2 mc^2}{kT_{recomb}} = \frac{3\alpha^2}{8}\lambda_{recomb} \qquad (6)$$

Ignoring dark matter and neutrinos (μ = ζ = 1), Equ.(2) gives $\xi_{\gamma N} = 680\lambda_{equal}$. Hence, using the hypothesis that $t_{equal} = t_{recombination}$, Equ.(2) with Equ.(5) give,

$$0.475(2+2x_1+x_1^2)e^{-x_1} = 1/\xi_{\gamma N} = 1.47\text{x}10^{-3}/\lambda \qquad (7)$$

which is an equation purely in λ as the unknown. Solving it gives $x_1 = 26.505$ and hence $\lambda = 1.33 \text{ x } 10^6$, and hence $\xi_{\gamma N} = 9 \text{ x } 10^8$. This is about half our best estimate ($1.9 \text{ x } 10^9$) and virtually the same as that obtained from the Section 2 'fiddle factor' method.

Alternatively, including both dark matter and neutrinos ((μ = 7.4, ζ = 1.681), Equ.(2) gives $\xi_{\gamma N} = 2994\lambda_{equal}$. Hence, using the hypothesis that $t_{equal} = t_{recombination}$, Equ.(2) with Equ.(5) give,

$$0.475(2+2x_1+x_1^2)e^{-x_1} = 1/\xi_{\gamma N} = 3.34\text{x}10^{-4}/\lambda \qquad (7)$$

which is an equation purely in λ as the unknown. Solving it gives $x_1 = 28.163$ and hence $\lambda = 1.41 \text{ x } 10^6$, and hence $\xi_{\gamma N} = 4.2 \text{ x } 10^9$. This is about double our best estimate ($1.9 \text{ x } 10^9$) and is again virtually the same as that obtained from the Section 2 'fiddle factor' method.

4. Conclusion

The time (temperature) at which the radiation era ends is defined by the equality of the radiation energy density and the matter density. However, this does not provide a unique definition because of the ambiguities regarding what is counted as radiation and what is counted as matter. Equality of electromagnetic radiation and baryonic matter occurs at t_{equal} = 615,000 years and 2130 K. However, if dark matter is included, equality occurs much earlier, at times of order 10,000 to 40,000 years.

Neither result is especially close to the time of recombination (185,000-363,000 years). Nevertheless, if the hypothesis is made that t_{equal} is the same as the time of recombination, the photon:baryon ratio can be estimated and found to be within about a factor of 2 of the observed value.