Appendix G
Gauge Theories and the Running Coupling Constants

§G.1
In the preceding Appendices we have given a very brief, indeed hopelessly inadequate, description of early versions of the theories of gravitation, electromagnetism, and the weak and strong nuclear forces. All these early theories are out of date. In all but one case, this leads to a significant change in the way in which we view the universal constants at the heart of the theories. Since the universal constants are our subject matter, we cannot entirely ignore this crucial fact. Consequently, in this Section, we shall attempt to explain the later theories a little. These theories are complicated. However, our objective is primarily to explain their impact on the nature of the coupling constants. Specifically, it turns out that the magnitudes of the coupling constants e, G_F and g_s depend upon the energy of the process involved. Fortunately we shall see that this energy dependence only becomes significant at energies greater than roughly 100 MeV. This means that, for times later than about a millisecond after the Big Bang, when typical thermal energies have fallen below ~30 MeV, the universal constants are independent of energy and take the values discussed above. In other words, the universal constants are indeed constant[1].

§G.2 Gravity
In the case of gravity, the theory presented in Appendix C is that of Newton. The 'later' theory is hardly recent. It is Einstein's General Relativity, which is approaching its centenary. General Relativity replaces the single gravitational potential of Newton's theory with a set of ten potentials. These ten potentials are independent, and hence ten separate field equations are required to determine them. Einstein's achievement was to find the right set of ten, monstrously non-linear, differential equations which have a certain crucial property: covariance. This property ensures that the algebraic form of the equations looks the same to all observers, even observers so perverse as to employ curved coordinate systems and to be in an arbitrary state of varying motion. Incidentally, the "Theory of Relativity" is rather a misnomer. A better name would be the "Theory of Objectivity". Einstein's aim was to find expressions for the laws of physics which look the same to all observers. This was inspired by a belief in the objective reality of the world. If the laws of physics are to express an objective fact about an objective world, then they cannot vary according to which observer is chosen to describe them.

Alone of the 'later' theories of the four forces, general relativity is of crucial importance at late times (i.e. after one millisecond – and indeed to the present day and beyond). However, it is also the only 'later' theory which introduces no change to the corresponding coupling constant. There is only one such constant in Einstein's theory, and it is the same as that in Newton's theory – the universal gravitational constant, G. Relativity makes no change to its magnitude. Nor does general relativity necessitate the introduction of additional constants. However, even within the constraint of covariance

[1] Unless, as some people believe, they have an additional explicit time dependence, not dictated by the theories explained here. This is discussed further in Chapter ?

there is scope for the introduction of a further term in Einstein's field equations which does introduce a new constant: the cosmological constant, Λ. This features as one of our universal constants (or rather a normalised form of it, Ω_{λ}).

However, gravity is also the only theory which has not (yet) been cast into a "quantum" form. If and when this is done, it may be that some change in the nature of G will result, e.g., an intrinsic energy dependence.

§G.3 The Strong Nuclear Force

In Appendix F we described the strong nuclear force as being a force between nucleons mediated by pions. In fact, the strong force does not only act between nucleons, it also acts between almost all of the particles in the particle zoo. Specifically, it acts upon the hadrons (see Appendix B for a description of the particle zoo). This is really a tautology because the hadrons are *defined* as being those particles which are subject to the strong force. But the large majority of particles are hadrons. The only known exceptions are, (a)the six leptons (the electron, the muon and the tau, and their corresponding neutrinos) and their antiparticles, plus, (b)the field quanta, or gauge particles, of the electroweak force: the photon and the W and Z bosons, (c)the Higgs particle (if it exists). There may also be other particles yet to be discovered which will not be subject to the strong force (e.g. WIMPs, gravtitons).

The second thing that is wrong with the description of the strong nuclear force in Appendix F is that it is not only mediated by pions. For one thing, it can be mediated also by other mesons, such as the rho, omega and sigma mesons, for example. And these mesons are all subject to the strong force themselves. This is very different from the situation for electromagnetism. Two charged particles are subject to the electromagnetic force mediated by photons. But two photons do not interact (directly).

As described in Appendix B, the quark model has provided a systematic way of classifying the zoo of hadrons – in much the same way that the electron orbital model systematises and explains the periodic table of the elements. This provides an alternative definition of a hadron: it is any particle composed of quarks. The theory of the strong force is now best regarded as a theory of interactions between quarks. The field quanta which mediate the strong force between quarks are a new addition to the zoo: the gluons. Despite being the fundamental constituents of the standard model of particle physics, neither quarks nor gluons have ever been observed directly. The reason for this is closely related to the principle subject of this section, namely the energy dependence of the strong coupling constant.

It turns out that, at large distances, the strong force between two quarks increases without limit. This is in stark contrast to the strong force between two nucleons. It is as if pairs of quarks are tied together with a piece of string. So long as the quarks are close enough together then the string is slack and the quarks can move around without interacting much at all. This is known as "asymptotic freedom". However, if you try to pull the quarks apart, the string stops them separating. This is known as "infrared slavery" or, more obviously, as "confinement". If you supply enough energy to break the string, then you get two new bits of string[2] – both of which have a quark at each end! For example, if you fire an energetic particle into a meson consisting of two quarks, and hit it hard enough to break it up, then instead of getting two free quarks you get two new mesons both comprised of a pair of quarks. In practice you'd get a whole potage of things, of which a second meson may be just one possibility – but free quarks are not part of the soup.

The theory which describes the quark-quark forces and their mediation by the gluons is Quantum Chromo-Dynamics, or QCD for short. The name derives from the equivalent quantum field theory for electromagnetism, i.e. Quantum Electro-Dynamics (QED). The prefix "Chromo" refers to the fact that the quarks and gluons come in various types which are distinguished by a label called their "colour". This is nothing whatsoever to do with what we normally mean by "colour", of course. All four of the fundamental forces, gravity, electromagnetism and the weak and strong nuclear forces, are mediated by types of field called "gauge fields". This means that the force fields are subject to a certain type of symmetry. Electromagnetism, when considered to act alone, is unique in being an abelian gauge theory. This means that the symmetry group underlying its gauge field involves commuting terms only (i.e. AB = BA). The consequence of this is that the quanta of its gauge field, the photons, do not interact with each other (directly). In contrast, all the other forces involve non-abelian (non-commuting) elements which give rise to field quanta which interact with each other. Thus, the W and Z bosons are subject to the weak force, two gluons exert a strong nuclear force on each other, and the energy density of the gravitational field will itself act as a source of gravity.

The mechanism which gives rise to confinement, asymptotic freedom and the energy dependence of the strong coupling constant is called "anti-screening". The phenomenon of screening is familiar from electricity and magnetism. If an electric charge is placed in a polarising medium, then, from a distance, the charge appears to be reduced in magnitude. The reason is that charges of opposite sign from within the medium are attracted to it. This has the effect of reducing the apparent magnitude of the charge when seen from outside the polarised medium in its vicinity. However, if you penetrate within the polarised region, the apparent size of the charge increases. Ultimately the true ('bare') magnitude of the charge is revealed at sufficiently short distances. The phenomenon of anti-screening is the opposite. In this case the equivalent of the polarising medium acts so as to amplify the 'charge' when seen from a distance. In the case of QCD, this comes about as a consequence of the three 'colours' of the gluons (as contrasted with just one type of electric charge). The part of the 'medium' is played by the vacuum itself, since, in quantum field theory, the vacuum is subject to a continual process of particle-antiparticle pair creation, and hence can act in a similar manner to a polarisable medium. The result is that a quark has its strong 'charge', i.e. its coupling constant g_s, amplified by a cloud of virtual gluons and virtual quarks. However, this amplification is neutralised if one can penetrate the cloud and get sufficiently close to the quark.

Thus, the consequence of the anti-screening is that g_s appears larger from greater distances and smaller when close in. The small coupling strength at small distances results in asymptotic freedom. At sufficiently large distances, the coupling strength

[2] These are not the strings of string theory. They are just an illustrative device.

increases without limit, and this produces confinement of the quarks. Also, in quantum mechanics, large distances align with low energies and small distances align with high energies. So, g_s is effectively larger at low energies and becomes smaller at high energies. So we see that the energy dependence of the coupling strength and confinement of the quarks are both facets of the same phenomenon, and they both result from the anti-screening by the gluons.

Quantum field theories are strange beasts. Traditionally they start by postulating how the energy density (or, more correctly, the Lagrangian) depends upon the fields in question. The energy density, or Lagrangian, also depends upon the coupling constant, as we see in the examples of Appendices E and F. A more or less standard mathematical procedure then permits predictions for physical processes to be deduced from the Lagrangian. All quantum field theories share a common feature: the result of this procedure is that all the predictions give an infinite answer! It is rather surprising then that these are the class of theories which have produced the most impressive agreement between theoretical prediction and experimental measurements in the whole of science, the record to date being in agreement to 9 decimal places[3]. How is such precision attained from the initial embarrassment of an infinite result?

The idea is that the constants which enter the Lagrangian, such as the coupling constants (e.g. charge) and the masses of the particles, are not the true physical values of these quantities. Rather they are some unknown 'bare' charges and masses. These are envisaged as being what the charge or mass would be if the particle could be stripped of its attendant cloud of virtual particles. Of course, this cannot be done and the bare values of the constants are unobservable. It turns out that, for a special class of theories, the troublesome infinite quantities of the theory can be combined with these bare charges or masses in a consistent manner – and the result redefined as the true physical charge or mass. The bare parameters gobble up the infinities and become physically real. This leaves the theory, rather miraculously, both finite and expressed in terms of the desired, real, physical parameters. The special feature which allows this spectacular trick to be performed is called "renormalisability". Quantum electrodynamics, quantum chromodynamics and the theory of the electroweak field are all renormalisable. Indeed, were they not, they would not be credible theories. The feature which leads to their being renormalisable is the fact that they are all gauge theories. Further details of these fascinating results is, regrettably, beyond the scope of both this book and its author.

The process of renormalisation effectively subtracts an infinite quantity from an infinite bare charge to leave a finite physical charge. The same renormalisation process can also lead to the charge becoming energy dependent. In practice, the techniques employed to study this are applicable only when the coupling constant is sufficiently small. This is because the method used is perturbation theory, in which quantities are expressed as a power series in the coupling constant. If the coupling constant is too large, such series

[3] Experiment and theory both give the normalised magnetic moment of the electron as g = 2.00233184, but disagree in the next (10^{th}) decimal place.

may be divergent[4]. Nevertheless, within a limited range of energies, such techniques provide explicit, if approximate, expressions for the variation of the coupling constant with energy. In the case of QCD, the result for the energy dependence of $\alpha_s = g_s^2 / 4\pi$ is as follows:-

$$\alpha_s(E) = \frac{4\pi}{\beta_0 L}\left\{1 - \frac{2\beta_1}{\beta_0^2}\cdot\frac{\log(L)}{L} + \frac{4\beta_1^2}{\beta_0^4 L^2}\left[\left(\log(L) - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right]\right\} \tag{G.1}$$

where $L = 2\log\left(\frac{E}{\Lambda}\right)$, noting that "log" denotes the natural logarithm, as usual, and E is the energy of the process being considered, and Λ is some parameter with the dimensions of energy. This parameter is called the "strong force energy scale". The parameters β_0, β_1, β_2 in Equ.(5.6.1) depend upon the number, n_q, of 'contributing' quarks. A quark will contribute if the energy, E, exceeds its mass. The parameters β_0, β_1, β_2 are given by,

$$\beta_0 = 11 - \frac{2}{3}n_q; \quad \beta_1 = 51 - \frac{19}{3}n_q; \quad \beta_2 = 2857 - \frac{5033}{9}n_q + \frac{325}{27}n_q^2 \tag{G.2}$$

Equ.(G.1) applies only for $E > \Lambda$. As E approaches Λ from above, L approaches zero, and the values of α_s and g_s diverge towards infinity. This illustrates the confinement property, i.e. that QCD becomes very strongly coupled at low energies. However, it is important to realise that it is not an accurate prediction that g_s diverges at the finite energy Λ. Rather, Λ marks the energy at which the perturbation expansion used to derive (5.6.1) ceases to be indicative. In other words, Equ.(G.1) only applies if E is sufficiently much greater than Λ. Nevertheless, Equ.(G.1) correctly indicates that the strength of the strong force increases at reducing energies.

Note that, for a given number of contributing quarks, specifying the strong energy scale, Λ, is sufficient to determine g_s at any required energy. Actually Λ, like the β-parameters, also depends upon the number of contributing quarks, n_q. Consequently it is better written as $\Lambda^{(n_q)}$. Within an energy range which does not change the number of contributing quarks, n_q, $\Lambda^{(n_q)}$ is constant. Thus, whilst g_s is no longer a constant, its place is effectively taken by $\Lambda^{(n_q)}$, which *is* constant – at least until another n_q boundary is passed.

Equ.(G.1) also illustrates the asymptotic freedom property, i.e. that $g_s \to 0$ as $E \to \infty$.

The masses of the quarks are known only very approximately (the reason being that they are never observed directly, so their masses must be deduced indirectly). The masses of the six quarks are roughly (in MeV):-

[4] The matter is more complicated than this really because the perturbation series of QFT are probable all divergent and should be understood as asymptotic expressions only.

U (up)	D (down)	S (strange)	C (charm)	B (bottom*)	T (top*)
3	6	110	1,250	4,300	174,000

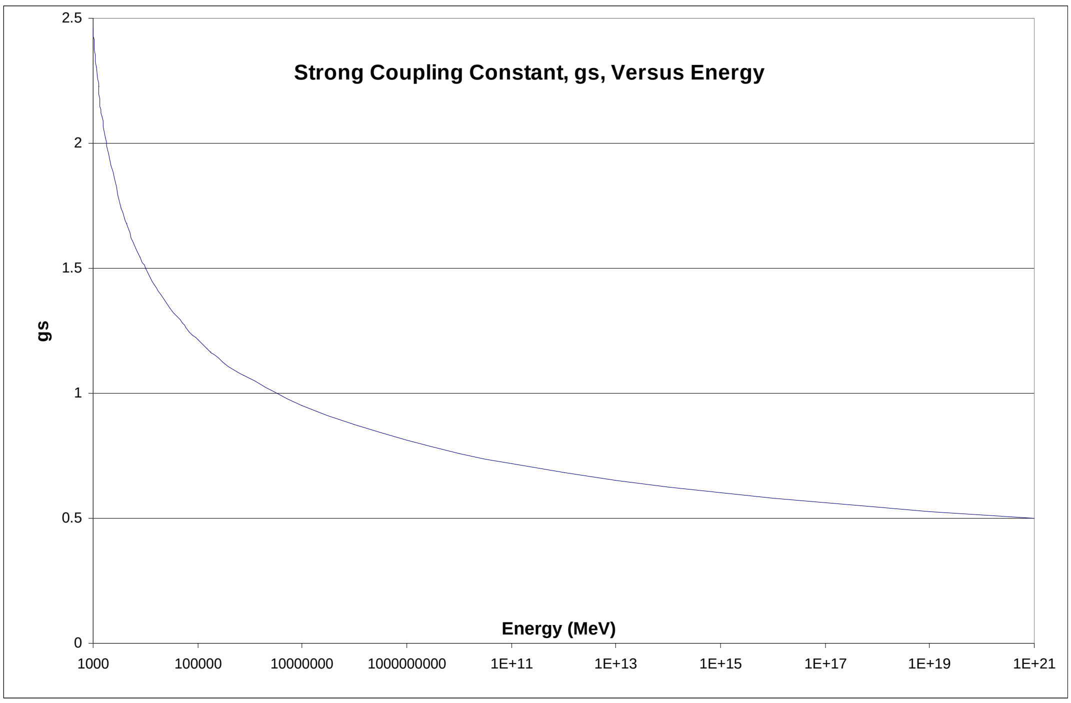
*these were once known as Beauty and Truth. It's a pity these names died out in my view.

Thus, for energies between 4,300 MeV and 174,000 MeV there are 5 contributing quarks, $n_q = 5$. Hence, $\beta_0 = 7.666$, $\beta_1 = 19.333$, $\beta_2 = 361.815$. In this energy range the experimentally determined strength of the strong force, g_s or α_s, corresponds to a value of the energy scale of $\Lambda^{(5)} = 216$ MeV. This allows us to plot α_s against energy in the range 4,300 MeV to 174,000 MeV. Outside this range, the value of $\Lambda^{(6)}$, $\Lambda^{(4)}$ and $\Lambda^{(3)}$ are found by ensuring that Equ.(G.1) gives a continuous result for α_s as the number of contributing quarks changes when their threshold energies are crossed. Thus we find, $\Lambda^{(6)} = 91$ MeV, $\Lambda^{(4)} = 298$ MeV, and $\Lambda^{(3)} = 336$ MeV.

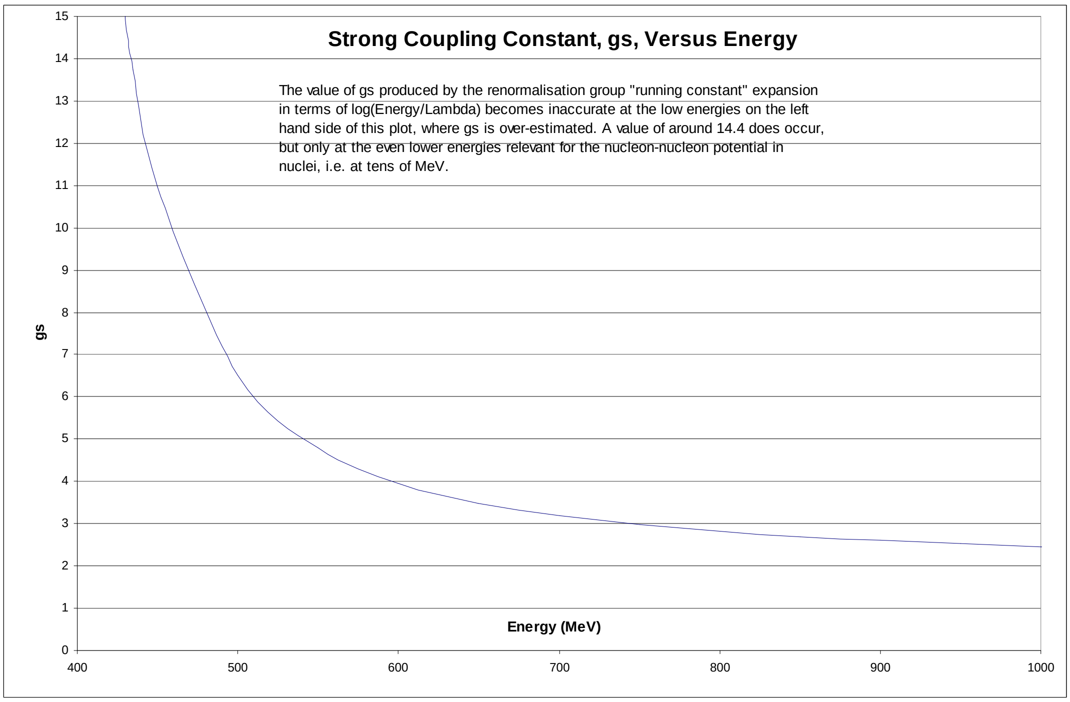
Note that the process cannot be continued to lower energies. When $n_q = 3$, and $\Lambda^{(3)} = 336$ MeV, the coupling constant diverges before the lower end of the energy range, the mass of the strange quark (110 MeV), is reached. The perturbation method breaks down at this point. In fact, it becomes inaccurate at rather higher energies.

At energies above 1 GeV, where the results are accurate, the coupling strength of the strong force varies with energy as shown in the following graphs. Specific values of interest are:_

At E = 642 MeV, gs = 3.545, $\alpha_s = 1.0$ (but not accurate here)
At E = Mz = 91 GeV, $g_s = 1.221$, $\alpha_s = 0.1186$
At E = 3100 GeV, $g_s = 1.0$, $\alpha_s = 0.07958$



Below 1 GeV the results become increasingly inaccurate, but can be extended down to about 435 MeV, at which energy g_s reaches the low energy value of ~13.5.



Note that the variation of gs is very marked, not a minor correction, even over an energy range which is easily achieved experimentally. Incidentally, experimental data between 2 GeV and 200 GeV does clearly display the variation of the coupling strength of the strong force with energy as expected from the above results, see for example Hinchcliffe (2005). For these reasons it is important to be aware of this energy dependence of the strong coupling 'constant' when dealing with energies of the order of 100's of MeV or more. However, recall that the thermal energies of the Big Bang fireball have already fallen to ~30 MeV after just one millisecond, so that the energy dependence of g_s is not important after this time[5].

In view of the fact that the above perturbation theory results are restricted to energies above about 430 MeV, the reader may wonder whether QCD is actually consistent with the low energy behaviour of the strong force, as exemplified by the inter-nucleon force in atomic nuclei. Addressing such questions requires a different mathematical method. The method which has been developed to cope with the challenge posed by QCD in the non-perturbative energy regime is lattice gauge theory. In this approach, the divergences of the continuum theory are removed from the start by working with a spacetime defined as a discrete set of points, i.e. a lattice. In principle, the lattice gauge formulation should solve all our problems! Unfortunately, it is computationally extremely demanding.

[5] At least, it is not important unless you are trying to discern extremely small changes in g_s – which is exactly what people hunting time dependence of the 'constants' may be trying to do.

Nevertheless, significant progress has been made in lattice gauge theory predictions. For example, Ishii et al (2006, 2009), Pennington (2009), Takahashi (2006, 2009), Bean 2001, 2002) and Machleidt (2000, 2007) have had at least qualitative success in deriving the form of the inter-nucleon potential from QCD, though there are still significant points of disagreement and uncertainty. Some authors demonstrate the strongly repulsive core of the potential at distances below ~0.5 fm, and also reproduces the qualitative shape of the attractive part of the potential. A quantitatively accurate calculation of the strong nuclear potential from QCD has not yet been achieved. However, Christine Davies (2006) has reported increasing success in applying lattice gauge theory to the evaluation of the masses of the hadrons.

G.4 Electromagnetism and The Weak Nuclear Force

The modern understanding of electromagnetism and the weak nuclear force is that they are aspects of the same underlying force. Just as special relativity combined electricity and magnetism into a single electromagnetic field, so electroweak theory combines electromagnetism with the weak nuclear force. The electroweak theory is a gauge theory and is renormalisable. In the case of electroweak theory we are on rather more secure foundations than the case of QCD. Unlike QCD, in which neither the quarks nor the gluons are directly observed, the corresponding particles in electroweak theory are both directly observed and very well characterised. The equivalent of the quarks in the weak force part of the theory are the leptons, i.e. the electron, the muon and the tau, plus their corresponding neutrinos, and their antiparticles (12 distinct particles in all - probably). The role of the force-carrying gluons, i.e. the gauge field quanta, is played by the W and Z bosons as regards the weak force, and by the photon for electromagnetism.

The adoption of this renormalisable gauge theory description of the electroweak force implies two things: firstly, that the electromagnetic coupling constant (e) and the weak coupling constant, G_F, are related, and, secondly, that they become energy dependent. The relationship between the electromagnetic and weak couplings is given by,

$$e = g \sin \Theta_W \quad \text{where,} \ \sin \Theta_W = \sqrt{1 - \frac{M_W^2}{M_Z^2}} = 0.4713 \qquad \text{(G.3)}$$

(For the derivation of this relation from the electroweak Lagrangian see Mandl and Shaw, 1993, Chapter 12). Here, g (without any subscript) is the weak coupling constant within the electroweak gauge theory, and e is the electric charge. They are related through the Weinberg angle, Θ_W, which is determined by the ratio of the W and Z masses (which is 80.425 GeV / 91.188 GeV). At low energies, the electromagnetic α is 1/137.036 so that the dimensionless charge is $e = 0.3028 = \sqrt{4\pi\alpha}$. Hence, at low energies the electroweak coupling constant is g = 0.6425. The old fashioned Fermi theory of the weak force is expressed in terms of the Fermi constant G_F, which is given in terms of g by,

$$G_F = \frac{g^2}{4\sqrt{2} \cdot M_w^2} \qquad \text{(G.4)}$$

For the derivation of this relation from the electroweak Lagrangian see Mandl and Shaw, 1993, Equations 12.49 and 11.71. Alternatively, see the "Review of the Electroweak Model" on the Particle Data Group website, Erler and Langacker (2005). Since the Fermi theory applies only at low energies, the low energy value for g is used in the above expression to give $G_F = 1.13 \times 10^{-5}\ \mathrm{GeV}^{-2}$ (which is in reasonable agreement with our previous value $1.02 \times 10^{-5} / M_p^2 = 1.16 \times 10^{-5}\ \mathrm{GeV}^{-2}$).

Considering higher energies leads to coupling constants, e and g, which *increase*. The situation is the converse of that for the strong interactions. In the case of the electroweak interaction, the existence of a cloud of virtual particles around a real particle with electric or weak charge leads to screening rather than anti-screening. Thus, from far away, the charge appears to be less than it does close up. At higher energies, therefore, deeper penetration into the virtual cloud towards the bare charge becomes possible with the result that the charge (i.e. the coupling constant) appears larger. The electroweak contribution to this effect is given by a perturbation theory expression as follows,

$$\Delta\alpha^{\mathrm{lep}} = \sum_l \frac{\alpha}{3\pi}\left\{2\log\frac{E}{M_l} - \frac{5}{3}\right\} \tag{G.5}$$

see, for example, Jegerlehner (2002). The significance of the superscript $^{\mathrm{leptonic}}$ will emerge below. The fine structure constant at the energy E is given in terms of its low energy value by $\alpha(E) = \dfrac{\alpha(0)}{1-\Delta\alpha}$.

Equ.(G.5) involves a sum over the massive leptons, i.e. the electron, the muon and the tau. It applies only when the energy is sufficiently larger than the lepton mass. As for the strong interactions, only leptons which are less massive than the energy in question contribute to the sum in Equ.(G.5). Thus for energies in excess of 1.777 GeV, all three massive leptons contribute, whilst for energies between 105.6 MeV and 1.777 GeV only the electron and the muon contribute. At energies below 105.6 MeV, only the electron contributes.

Thus, for example, at the Z mass of 91.188 GeV, $\Delta\alpha^{\mathrm{lep}} = 0.03142$, so that this correction to the fine structure constant produces $1/\alpha = 132.730$, compared with the low energy value of $1/\alpha = 137.036$. At only 100 MeV, a rather modest energy, we find $\Delta\alpha^{\mathrm{lep}} = 0.0082$, so that this correction to the fine structure constant produces $1/\alpha = 135.916$. So the correction is significant, i.e. of order ~1%, even at these low energies.

However, the situation is more complicated than this, for two reasons. The first is that the cloud of virtual particles includes particles which are subject to the strong force as well as the electroweak force. Consequently, there is also a hadronic correction to the electroweak coupling constants. These virtual hadrons also cause screening and hence will also cause the electroweak coupling constant to increase at higher energies. The underlying theory is beyond our scope [but see, for example, Erler and Langacker (2005), Jegerlehner (2002), and Narison(2001)]. At energies equal to the Z mass, 91.188 GeV, the hadronic correction is comparable to the electroweak correction, specifically $\Delta\alpha^{had}$ = 0.02791. Thus the total correction at this energy is $\Delta\alpha^{total}$ = 0.02791 + 0.03142 = 0.05933. This would give 1/α = 128.906 at the Z mass.

However, there is another subtlety. This is related to the fact that the variation of the coupling constants with energy is, in a sense, a mere matter of convention. We could consistently use constants which did not vary with energy at all – so long as we appropriately adjust the predictions for all physical quantities accordingly. For example, the anomalous magnetic moment of the electron expressed as a function of the fine structure constant, α, will be a different function depending upon whether we have adopted a convention in which α is rigorously constant or one in which α varies with energy. Even within schemes which considered the coupling constants to vary with energy, there are different conventions possible regarding the details of the way in which the theory is developed, i.e. just how predictions for physical quantities are made. Two such options are the "on-mass-shell" and "minimal subtraction" schemes. Which scheme is adopted can affect the values attributed to, say, the quark masses. The correction factor for α given above, $\Delta\alpha^{total} =$ 0.05933, applies in the "on-mass-shell" scheme. However, the convention adopted by the Particle Data Group is the "minimal subtraction" scheme. There is an additional correction to apply to convert from one scheme to the other, namely,

$$\Delta\alpha^{scheme} = \frac{\alpha}{\pi}\left(\frac{100}{27} - \frac{1}{6} - \frac{14}{4}\log\left(\frac{M_z}{M_W}\right)\right) = 0.0072 \qquad (5.6.6)$$

Adding this correction to the total gives $\Delta\alpha^{total} =$ 0.06652 (when E = M_Z). Thus the (reciprocal of the) fine structure constant in the "minimal subtraction" scheme, and at an energy equal to the Z mass, is (1 – 0.06652)*137.036 = 127.918, which agrees with the values quoted in Erler and Langacker (2005) and in Tegmark, Agirre, Rees and Wilczek (2006), Table 2.

Converting to 'e' and dividing by the sin of the Weinberg angle gives the value of the weak coupling constant g at E = M_Z, i.e. 0.3134/0.4713 = 0.6650. This disagrees with the value given in Tegmark, Agirre, Rees and Wilczek (2006), Table 1 (which also seems to give the wrong value for the Weinberg angle). Possibly I am incorrect in assuming that the relation $e = g\sin\Theta_W$ continues to hold, with a constant value for the Weinberg angle, when energy corrections to the coupling constant are applied in the MS scheme. In other words, it may be that g has to be corrected for energy in a manner independent from that for e, and hence that the Weinberg angle changes. However, Tegmark, Agirre, Rees and Wilczek (2006), Tables 1 and 2 look inconsistent whatever interpretation one applies.

A plot of the running of α due both to the leptons and the hadrons is given in Jegerlehner (2002).

Finally, we note that the dependence of α on energy has been observed directly in the OPAL collaboration experiments on Bhabha scattering, specifically within the energy range ~1.8 Gev to ~6 GeV.

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