Appendix F
The Strong Nuclear Force Coupling Constant, g_s

The original motivation for postulating the existence of the strong nuclear force was to explain how atomic nuclei were held together against the electrostatic repulsion between the protons. Historically, the first approach was to describe the interaction between two nucleons in terms of a potential, $V(\bar{r})$. The strong nuclear force is of short range, though not so short in range as the weak force. The strong force falls off rapidly after distances of the order of the nuclear size, typically ~1.4 fm. So $V(\bar{r})$ must reduce to a very small value after distances of this order. We have already seen, in Appendix E, what might cause a rapid reduction in an inter-particle potential. When the potential is mediated by a force field and this force field has quanta which have non-zero mass, then the potential will be of Yukawa form, Equ.(E.13).

Indeed, this application was the original suggestion of Yukawa, who postulated the existence of a particle to act as the mediator of the strong nuclear force. Knowing the range of the nuclear force (X ~ 1.4 fm), Yukawa could estimate the required mass of this field quantum. i.e. $M = \hbar / Xc$ = 197.6 MeV.fm / 1.4 fm ~ 140 MeV. A particle of this mass, the pion, was duly discovered. So the range of the strong nuclear force, and the size of atomic nuclei, are the result of the pion having the mass that it does.

The approximate magnitude of the strong nuclear potential can be estimated by requiring that it explain the stability of the atomic nuclei, starting with the simplest – deuterium. Moreover, we can attempt to reproduce the binding energy of the deuteron. This has been developed in detail in the "Cosmic Coincidences" Chapter 9b (The Binding of the Deuteron").

It is worth noting the great difference between the mass of the pion and the mass of the W and Z particles which mediate the weak nuclear force. The pion actually comes in both a neutral form of mass ~135 MeV, and a charged form of mass ~140 MeV. The W and Z have masses of 80,425 MeV and 91,188 MeV respectively - about 600 times greater than the pions. As noted above, this means that the weak force is of far shorter range than the strong force. It also means that, at distances of the order of a fermi, the weak force is very weak, due to the exponential dependence of the Yukawa potential, above. In fact, the weakness of the weak force is almost entirely attributable to the large mass of the W and Z quanta which mediate the weak force. The intrinsic strengths of the strong and weak nuclear forces are not so different.

Thus, it was appreciated very early that the strong force was mediated by another particle, a field quantum. In analogy with the highly simplified description of the weak force, we can also describe the strong force very crudely in terms of an energy density, thus,

$$L = g_s \sqrt{\hbar c}\, \rho(\bar{r}) \phi(\bar{r}) \qquad \text{(F.1)}$$

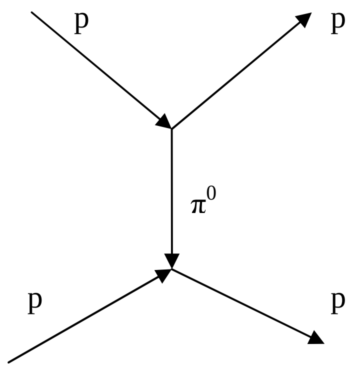
where g_s is the strong coupling constant (analogous to electric charge), ρ is the density of nucleons and ϕ is the field describing the pions. Rather more correctly, the early theories used an interaction between the nucleons and the pion field described by quantum fields in the form of an energy density (Lagrangian),

$$L_I = i g_s \sqrt{\hbar c} \cdot \Phi_j \overline{\Psi} \gamma_5 \tau_j \Psi \qquad \text{(F.2)}$$

The factor of $\sqrt{\hbar c}$ has been inserted into (F.1, 2) to make the strong coupling constant g_s, dimensionless. [This may be confirmed by noting that the pion field has dimensions of $(\text{energy/length})^{0.5}$, and , of course, the dimensions of the nucleon density, $\rho \sim \overline{\Psi}\gamma_5\Psi$, are volume^{-1}. The pion field dimensions are specified by the requirement that the square of their gradient must be an energy density].

As explained in Appendix E, this is (roughly speaking, and using the language of classical physics) the energy required for a nucleon to create or absorb a pion, per unit volume. In quantum field theory, this energy of interaction can be related to the probability of a pion being created or annihilated. To within numerical factors, dividing by the magic $\hbar$ turns the interaction energy density into a probability[1] for pion creation or annihilation per second per unit volume.

But such events are actually impossible. Starting with a nucleon, the creation of a pion whilst leaving a nucleon still present violates the conservation of energy. However, in quantum field theory, such a violation can occur but only for a very short time. The pion which is created is said to be “virtual” since its energy and momentum do not have values which are possible for a real, physical pion. This virtual pion must very soon be annihilated in a similar but reversed interaction. So, both the initial and final states involve simply two nucleons and nothing else, but the momentary exchange of the pion causes a ‘force’ to act between them. This process is specified pictorially as a Feynman diagram. For example, the force between two protons due to the exchange of a neutral pion has the Feynman diagram,



Algebraically, the significance of this is that it involves two ‘vertices’. That is to say, there is a vertex at which a pion is created and also one at which the pion is annihilated.

[1] Strictly, a complex valued probability amplitude – which is a purely quantum mechanical concept and is roughly the square-root of the probability.

Both of these vertices have probabilities[1] of occurring proportional to the interaction energy given by (F.1, 2). The overall interaction potential due to the process represented by the above Feynman diagram is therefore proportional to the square of (F.1, 2). In particular this means that the inter-nucleon potential is proportional to the square of the strong coupling constant, g_s. The mathematical machinery of quantum field theory can be used to derive the effective inter-nucleon potential corresponding to the above Feynman diagram. It turns out to be[2],

$$V_s(r) = -2 \cdot \frac{g_s^2}{4\pi} \cdot \left(\frac{M_\pi}{2M_N}\right)^2 \cdot \frac{e^{-M_\pi r}}{r} \qquad \text{(F.3)}$$

where M_π is the mass of the pion (135 or 140 MeV), and note that the negative sign indicates an attractive potential. For the low energies which are relevant in the later Chapters, in the order of tens of MeV or less, the coupling constant of the strong force, g_s, is roughly 13.5. By analogy with the electromagnetic fine structure constant, a quantity,

$$\alpha_s = g_s^2 / 4\pi = 14.4 \qquad \text{(F.4)}$$

is also widely used as a measure of the strength of the strong force, and known as the "strong fine structure constant" or simply the "strength of the strong force". This gives the amplitude of the Yukawa potential to be about 22 MeV, i.e.,

$$V_s(r) = -22\text{MeV} \times \frac{e^{-M_\pi r}}{M_\pi r} \qquad \text{(F.5)}$$

Note that in (F.3,5) we are using units with $\hbar = c = 1$.

What needs to be stressed is that this whole development of the strong force is extremely approximate and intended only to give a feel for its strength. Even at low energies, the pion exchange picture of the nuclear force is not at all adequate really. And at higher energies it is found that the actual field quanta of the strong force are not pions at all, but gluons. This will be discussed briefly in Appendix G.

In passing we note that the strong force is independent of the electrical charge. That is, the strong force between two neutrons is the same as that between two protons or between a proton and a neutron. In particle physics, the differently charged versions of essentially 'the same' particle type are distinguished by an abstract quantity called isotopic spin, or isospin for short. Thus, the proton and the neutron are different isospin states of a nucleon, and the neutral and charged pions, π^0, π^+, π^-, are different isospin states of "the pion". A group algebra is associated with isospin in analogy with ordinary spin, thus providing a means of rigorously specifying what it means for an interaction to be isospin independent. Providing that our interaction energy (Lagrangian) is chosen to be isospin independent then the charge independence of the strong force follows.

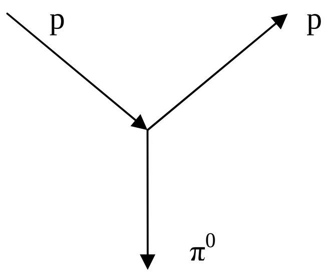
An effective coupling constant can be defined for each of the 'creation' interactions,

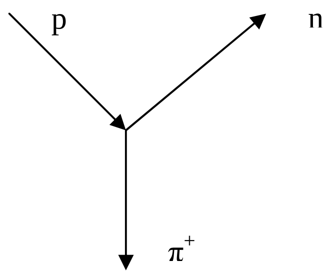
[2] Strictly, this applies to nucleons in so-called S-wave states, i.e. which are not rotating around each other.

Appendix F - The Strong Nuclear Force Coupling Constant, g_s

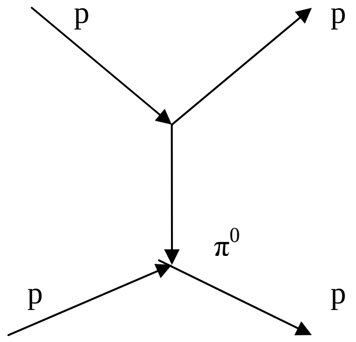
$$p \rightarrow p + \pi^0 \quad \text{and} \quad p \rightarrow n + \pi^+$$

In Feynman diagram terms these would be written,



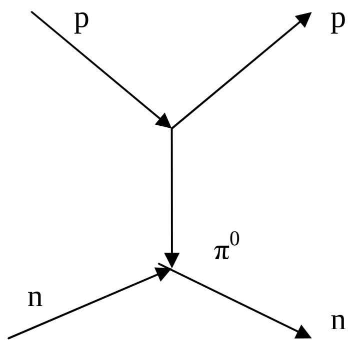


In turns out that to make the interactions isospin independent requires that the effective coupling constant for these two interactions, or 'vertices', must be related by, $g_{pn\pi^+} = \sqrt{2} g_{pp\pi^0}$. There is an amusing brain teaser regarding how this results in charge independence of the nuclear force. The 'force' between two protons is given, in lowest order, by the Feynman diagram,

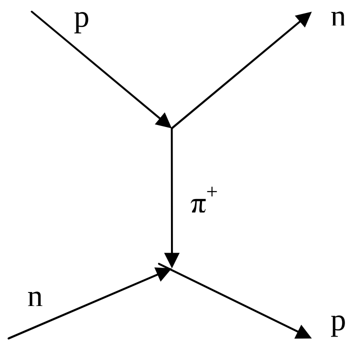


which gives a force proportional to $\left(g_{pp\pi^0}\right)^2$. However, the force between a proton and a neutron is due to the combined effect of two Feynman diagrams,

Appendix F - The Strong Nuclear Force Coupling Constant, g_s



Plus,



(Note that a diagram with an upward travelling negative pion is not distinct from the above diagram and should not be counted separately).

Naively one might thing that the combined probability amplitude for a pn interaction from the above two diagrams would be proportional to $\left(g_{pn\pi^+}\right)^2 + \left(g_{pp\pi^{0^2}}\right)^2$ and hence proportional to $3\left(g_{pp\pi^{0^2}}\right)^2$ (because $g_{pn\pi^+} = \sqrt{2}g_{pp\pi^0}$). The pn force is apparently 3 times larger than the pp force! The fallacy is that account has not been taken of the relative sign between the two Feynman amplitudes (or, more generally, their relative phase). Actually, because we are dealing with fermions, the sign of the contribution from the two diagrams is opposite. The pn scattering total amplitude is therefore actually proportional to $\left(g_{pn\pi^+}\right)^2 - \left(g_{pp\pi^{0^2}}\right)^2 = \left(g_{pp\pi^{0^2}}\right)^2$, so the pp and pn forces are indeed the same.

Another source of potential confusion for the novice is that the differential scattering cross-section for pn scattering is only half that for pp or nn scattering. This can also give the false impression that the nuclear force is charge dependent. In fact, if the differential cross-sections are integrated to give the total cross sections, all three processes, pp, nn and np, are the same[3]. The reason is that, in the case of pp and nn scattering, the identical nature of the particles means that scattering through an angle of θ is indistinguishable from scattering through an angle of $\pi - \theta$, so the integration is carried out over the interval $[0, \pi/2]$ only. For pn scattering, in contrast, since the particle are different, the integration is carried out over the full interval, $[0, \pi]$.

Finally we note that there is a crucial ingredient missing in the description of the inter-nucleon force by a Yukawa potential alone. This is the repulsive core. At distances less than ~0.5 fm the potential increases steeply, becoming large and positive – corresponding to a repulsive force. This rapidly increasing, large positive core to the potential is essential in explaining several features of the inter-nucleon force. The first is the fact that nuclear binding energies 'saturate'.

Consider a large nucleus consisting of many nucleons, N. Does every nucleon interact with every other equally? If so, because the number of pairs of nucleons is N(N-1)/2, the total binding energy would increase according to this factor (which is roughly proportional N^2 for large N). This is how the electrostatic Coulomb energy between the protons behaves (but with N replaced by Z, the number of protons). However, the nuclear force does not behave like this. The nuclear force is more analogous to the bonds between atoms which hold a molecule together. The nucleons tend to interact only with their nearest neighbours. Within the volume of a large nucleus, the number of nearest neighbours is just a fixed number (e.g. 12). So the binding energy will increase proportionally as 12N, i.e. proportional to N *not* N^2. The nuclear force is said to 'saturate', meaning that the force does not extend beyond the nearest neighbours.

Now the inter-nucleon force's repulsive core is essential in causing saturation. The strong repulsive core means that, when sufficiently close, the nucleons behave like hard, incompressible billiard balls. Had there been no such repulsive core, there would be nothing to stop the nucleons overlapping and, at least partially, occupying the same space. They would then all be able to get close to every other nucleon, with the result that the nuclear binding energy would increase according to N(N-1)/2. The property of saturation is essentially what leads to there being only a relatively small number of different chemical elements. The reason is that the Coulomb repulsion, which increases as Z(Z-1)/2 will inevitably become larger than the nuclear attraction for sufficiently large Z, since the nuclear force increases only as N (and we are assuming that Z is roughly N/2). Thus, the fact that there are roughly 100 chemical elements is evidence which points to the necessity of a strong repulsive core in the inter-nucleon force.

There are other phenomena in astronomy which require the inter-nucleon force to have a strong repulsive core. We mention in passing that these include, (a)the upper limit to the mass of neutron stars (the Tolman, Oppenheimer, Volkoff limit, which is reasonably well established observationally), and, (b)the mechanism underlying the massive Type II supernovae explosions. The latter is discussed in more detail in Chapter 5 and Appendix I of the "Cosmic Coincidences".

[3] That is, the same as regards the strong nuclear interaction evaluated at lowest order. Physical scattering of nucleons off nucleons is actually dominated by the electromagnetic interaction at low energies.