Appendix C
Newton’s Gravitational Constant, G

Written 2007: Reviewed 15/11/09

§5.2 Newton’s Universal Gravitational Constant, G

In Newton’s theory, the gravitational force between two point masses M_1 and M_2 is given by,

$$F = -G\frac{M_1 M_2}{r^2} \qquad \text{(C.1)}$$

where r is the distance between the masses. This effectively defines the universal gravitational constant G. The minus sign indicates that the force due to the gravitational field of particle 1 acting on particle 2 is in the opposite direction to the vector from 1 to 2. In other words, the gravitational force is attractive. Expressing the gravitational “force” field as an energy, as discussed in Appendix B, the gravitational potential energy is,

$$\text{P.E.} = -G\frac{M_1 M_2}{r} \qquad \text{(C.2)}$$

In classical mechanics, the force is the spatial derivative of the potential energy, but acting in the direction of decreasing potential, i.e.,

$$F = -\frac{d(\text{P.E.})}{dr} \qquad \text{(C.3)}$$

So we see that Equ.(C.1) and Equ.(C.2) are consistent, including the minus sign. [NB: This is because the derivative of $1/r$ is $-1/r^2$]. We labour this point somewhat for a good reason: the fact that the gravitational force is *attractive*, and therefore that the gravitational potential is *negative*, is the most important feature of the universe.

If the gravitational force had been repulsive, the constituents of the Big Bang fireball would simply accelerate away from each other forever. No structure would ever form in the universe. The attractive force of gravity is essential to bring about local reversals in the general tendency for the matter of the universe to fly apart. Such local reversals provide the mechanism for the creation of galaxies and stars and planets and life. More fundamentally, had gravity been attractive, the universe would probably not have formed at all. This is perhaps the most stunningly amazing feature of the universe: it all amounts to nothing.

Ask yourself, “what is the total mass-energy of the universe?” Think of this as the total energy of the universe with each mass m contributing mc^2 to the energy. The matter of the universe, plus its kinetic energy, and the electromagnetic radiation, all contribute to the “credit” column of the universe’s balance sheet. But gravity, having negative potential, contributes only to the “debit” column. The creation of the universe appears to have consisted of separating the positive and negative energies, with a net sum of close to zero, perhaps precisely zero. Guth has called it the ultimate free lunch. The possibility of the universe coming into being at all may be intimately connected with the net energy being zero.

But now back to Newton’s constant. The magnitude of G is 6.67 x 10^{-11} $kg^{-1}m^{3}s^{-2}$. Using Equ.(C.1) the reader may check that the Earth of mass 5.977 x 10^{24} kg and radius 6.37 x 10^{6} m results in a force acting on a 1 kg mass on its surface of 9.82 Newtons, i.e. the acceleration due to gravity at the Earth's’surface is 9.82 ms^{-2}, a familiar result.

Perhaps less familiar is the potential energy of this 1 kg mass on the Earth’s surface. Using Equ.(C.2) shows this to be a sizeable –6.3 x 10^{7} Joules, noting the minus sign of course. In school physics problems, gravitational potential energies are most often evaluated from some arbitrary reference position, such as the floor, rather than as absolute values. It comes as a surprise, perhaps, that the absolute potential energy is so large. The fact that the potential energy is both large and negative is the reason why it is not easy to leave the planet Earth.

Despite our observation that the gravitational potential energy of a 1 kg mass is rather larger than one might have thought, actually gravity is a very weak force. It does, after all, take the whole of the planet Earth to bring about that 6.3 x 10^{7} Joules of potential energy. Put even more graphically, it takes the whole of the planet Earth to generate one’s weight – a measly 78 kg-force in my case[1]. The relative weakness of gravity compared to the other forces is illustrated in the following Sections.

Finally we note that general relativity, which is also a theory of gravity, does not require the introduction of any further universal constants to describe the gravitational effects caused by matter. The same constant, G, appears in general relativity. There is potentially one other universal constant in general relativity, however. This is the so-called cosmological constant, Λ. This is discussed further in Chapters 5B and 5C of the Cosmology Tutorial. We note here only that the cosmological constant does not affect the gravitation caused by a given mass. Rather it is an additional, universal, source of gravity – and a potentially repulsive one at that.

---

[1] Regrettably this is *not* a universal constant and appears to be increasing.