Chapter 9D: Is There An Upper Bound To The Strong Force?

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1. Introduction

In Chapter 9C of the “Critique” we considered relatively small increases in the strong force, increasing g_s by 10% - 40%. This is sufficient to bind the diproton. However, we showed that, contrary to the claims commonly made, no diprotons would be formed in the Big Bang. Moreover, we argued that stars which could sustain life on planets would still be possible, though different from those of our universe. In Chapter 9 of the “Critique” we showed that a sufficient *reduction* in g_s would lead to a universe with no stable nuclei, and hence no chemistry. However we did not identify an upper bound for g_s in Chapter 9. The oft claimed upper bound based on diproton stability has been shown to be spurious in Chapter 9C. So we must ask - is there an upper bound on gs for a complex universe?

In this Chapter we make the observation that if g_s is increased by more than ~40% then deuterium is stable before ~1 second. This is the period when the leptonic reactions which interconvert protons and neutrons are still active. Thus, at small fractions of a second, the proton:neutron ratio will be less than the 87%:13% level that it attains prior to nucleosynthesis in this universe (at around 2 minutes or so). When deuterium becomes stable, the existing free neutrons are rapidly captured by protons to form deuterium, and subsequently helium. The hydrogen fraction of the primordial universe is determined simply by how many protons are left over. Thus, if deuterium is stable when the proton:neutron ratio is close to 50%:50%, then the universe becomes virtually all helium and no hydrogen. This is exactly the catastrophic scenario that other authors had envisaged as resulting from diproton stability – and which we have discredited. But, for increases of g_s sufficiently greater than 40%, there is potentially a mechanism resulting in an all-helium universe quite independent of diproton stability. In this Chapter we investigate the credibility of this mechanism.

2. The Proton:Neutron Ratio Before 1 Second – Reprise

We have derived this ratio in Chapter 6 of the “Tutorial”. It depends simply upon the neutron-proton mass difference compared with the temperature. We find:-

t (sec)	**T (^{o}K)**	**N:P**	**N (%)**	**P (%)**
0.001	2.31E+11	0.94	48.38	51.62
0.01	7.30E+10	0.81	44.88	55.12
0.05	3.26E+10	0.63	38.71	61.29
0.1	2.31E+10	0.52	34.30	65.70
0.2	1.63E+10	0.40	28.52	71.48
0.4	1.15E+10	0.27	21.42	78.58
0.5	1.03E+10	0.23	18.95	81.05
0.75	8.43E+09	0.17	14.44	85.56
1	*7.30E+09*	*0.13*	*11.36*	*88.64*

where we have used for the temperature,

$$T(^0K) = \frac{0.73 \times 10^{10}}{\sqrt{t(secs)}} \qquad (1)$$

Note that the p:n ratio does not depend upon the strength of the weak force, provided that equilibrium of the leptonic reactions prevails at every time shown. The strength of the weak force determines only the time at which freeze-out of the leptonic reactions occurs. Beyond this time there is no further change in the p:n ratio (except via the relatively slow beta decay of the neutron). Hence, the strength of the weak force will not affect anything we deduce in this Chapter *unless* it were reduced so markedly that the leptonic reactions were frozen-out at the times we consider. We will say a little more about this later.

3. Deuteron Binding Energy Versus Time For Stability

Again, we have shown in Chapter 6 of the “Tutorial” how the time & temperature at which deuterium becomes stable against photodisintegration can be found in terms of its binding energy. In this universe (B = 2.22 MeV) the answer is a little more than 2 minutes. More generally we have,

$$\left(2 + 2x_1 + x_1^2\right)e^{-x_1} < 2.0 \times 10^{-10} \qquad (2)$$

where $x_1 = B/kT$. The solution of (2) is $x_1 > 29.145$. Strictly, the number on the RHS of (2) will be different at different p:n ratios. However, the solution for ‘x’ is insensitive to, say, a factor of two change in the RHS, due to the dominant influence of the exponential. Thus, for RHS from 1×10^{-10} to 4×10^{-10} we have x in the range 28.45 to 29.84. Using (1) for the temperature we thus find the required deuterium binding energy for stability at various times as follows:-

time (sec)	**temperature (10^{10} K)**	**B (MeV)**	g_s / g_s^{nom}
1	0.73	18.3	1.34
0.1	2.31	58	1.8
0.01	7.3	183	2.7
0.001	23.1	580	4.3

4. Binding Energy Versus gs

For the binding energy of the deuteron we use a square well triplet potential of depth V = 35.1 MeV and with a = 2.054 fm. This reproduces B = 2.22 MeV. The potential well depth, V, is scaled by $\left(g_s / g_s^{nom}\right)^2$. This produces the following results:-

g_s / g_s^{nom}	B (MeV)
1	2.22
1.2	9.90
1.3	15.5
1.4	22.2
1.75	53.0
2.5	154
2.6	171
4	486
4.2	543

The values in the Table of Section 3 are obtained from those above by interpolation.

The dineutron binding energy is found in the same way using a singlet potential of V = 11.8 MeV with a = 2.8 fm, or alternatively using V = 16.1 MeV with a = 2.4 fm. The diproton binding energy is estimated from that of the dineutron by subtracting the Coulomb energy (0.6 and 0.5 MeV respectively). Thus,

g_s / g_s^{nom}	B_{nn} (MeV)	B_{pp} (MeV)
1.2	0.6 – 0.8	0 – 0.3
1.3	1.6 – 2.2	1 – 1.7
1.4	3.1 – 4.2	2.5 – 3.7

5. Deuteron-Lepton Reactions?

We are considering a scenario in which all the neutrons are converted to deuterium. But does this mean that the neutrons are safe from being converted back into protons by the ambient leptons? This is an issue that we do not have to face in our universe because the leptonic reactions are frozen out long before deuterium becomes stable. Happily it is an easy concern to dismiss. We will be considering times at which deuterium is stable against photodisintegration. Now, the number density and energy spectrum of the neutrinos and the electrons/positrons is essentially the same (modulo factors of order unity) as that of the photons. Thus, stability of the deuterons against leptonic reactions is assured by virtue of the relative weakness of the leptonic reactions compared to the electromagnetic reactions. Hence, we need not be concerned about reactions like $D + e \rightarrow (nn) + \nu_e$ or $D + e^+ \rightarrow {}_2^2He + \overline{\nu}_e$ or $D + \overline{\nu}_e \rightarrow (nn) + e^+$ or $D + \nu_e \rightarrow {}_2^2He + e$.

6. Dineutron or Diproton Formation?

Since we are considering such large increases in g_s, it follows that dineutrons and diprotons are very stable. However, we shall consider values of g_s / g_s^{nom} which are just sufficient to stabilise deuterium against photodisintegration at some time t_0. Because the binding energies of the dineutron and diproton are substantially smaller than that of the deuteron[1], they will be unstable to photodisintegration at time t_0. However, in principle they might form later – at lower temperatures. But inspection of the reaction rates shows that the nn and pp capture reactions are frozen out after about 1 millisecond. Consequently, these reactions do not influence the outcome either.

7. A Universe Without Hydrogen?

time (sec)	**temperature (10^{10} K)**	**B (MeV)**	g_s / g_s^{nom}	**N (%)**	**P (%)**	**P%-N%**
1	0.73	18.3	1.34	11.36	88.64	77.28
0.1	2.31	58	1.8	34.30	65.70	31.4
0.01	7.3	183	2.7	44.88	55.12	10.24
0.001	23.1	580	4.3	48.38	51.62	3.24

The above Table, combined from those of Sections 2 and 3, shows the fraction of free protons remaining after deuteron (and thence helium) formation, for increasing values

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[1] We are assuming, of course, that the triplet and singlet potentials both scale as $(g_s / g_s^{nom})^2$

of g_s. This Table assumes that the p-n capture reaction is sufficiently fast to mop up all the neutrons in the very short time available. In the case of the last line, this requires reaction times of a few milliseconds at most.

In fact, the n-p capture reaction *is* sufficiently fast. It's rate is 43,700 s^{-1} per mole/cm^3, independent of temperature. The reaction rate nevertheless increases as we go back in time because of the increasing particle densities. The reaction times are as follows:-

t (sec)	**T_R, reaction time (10^{-6} sec)**	**T^R / t**
1	6600	0.0066
0.1	200	0.0020
0.01	7	0.0007
0.001	0.2	0.0002

Thus, the reaction is expected to proceed essentially to completion in a very small fraction of the instantaneous time, t.

So, if g_s is increased by a factor of 3 or 4 or so, the amount of hydrogen in the universe will be dramatically reduced. For g_s / g_s^{nom} = 4.3 the universe will only be ~3% hydrogen (by mass), as opposed to 75% in this universe. Is this fatal to life? It is difficult to know for sure. 3% of the universe's mass is still a lot of hydrogen! Is it enough to provide the chemical conditions necessary for the evolution of life? In our present state of knowledge, it is not possible to say. Stars would be very different in such a universe. There is far less hydrogen to burn in the initial phase (which we can hardly call the main sequence in this alternative universe). And this hydrogen would burn via the strong p-p capture reaction to form diprotons, rather than by the usual weak interaction to deuterium. The stars would need to be cooler and less dense in the hydrogen burning phase in order to avoid being explosive (as described in the "Critique" Chapter 9C).

8. An All-Helium Universe?

Having established that, for g_s / g_s^{nom} > 3 or so, most of the nucleons are fused into deuterium, does the early nucleosynthesis stop there or is helium also formed during the leptonic period? The answer is that helium-3 and then helium-4 will form extremely rapidly. To show this, it suffices to consider the reactions,

$$D + D \to {}^3_2He + n \tag{3}$$

$${}^3_2He + {}^3_2He \to {}^4_2\ He + 2p \tag{4}$$

Hoffman et al give the reaction rates for these reactions only up to a temperature of 10^{10} K, namely roughly 3 x 10^8 s^{-1} per mole/cm^3 for both reactions. We actually want the reaction rates at between 2 x 10^{10} K and 23 x 10^{10} K (for times 0.1 sec and 0.001 sec respectively). It is likely that the reaction rates are at least an order of magnitude larger at these temperatures. However, we shall assume the slower rates at 10^{10} K for conservatism. The initial rate at which helium-3 is formed by reaction (3) is,

t (sec)	ρ_D (mole/cm^3)	$\dot{\rho}_{He3}$ (mole/cm^3)	time to exhaust deuterium (sec)
0.1	0.1	~3×10^6	~2×10^{-8}
0.01	3	~3×10^9	~10^{-9}
0.001	110	>3×10^{12}	~2×10^{-11}

The last column is a rough estimate of the time require to exhaust the initial inventory of deuterium, assuming the initial rate of its usage continues (note that the rate of usage of deuterium is twice the rate of helium-3 formation). Of course the rate of usage slows down as the deuterium is depleted. However, it is clear that essentially all the deuterium will be converted to helium-3 extremely quickly (i.e. in small fractions of a microsecond). Since the rate of reaction (4) is virtually the same, it follows that as soon as any concentration of helium-3 builds up, it will be immediately converted into helium-4, again in fractions of a microsecond.

Do the nuclear fusion reactions continue on past this point, creating heavier nuclei? This is difficult to be sure about. For one thing, it is not clear whether lithium-5 and/or beryllium-8, nuclei which are unstable due to fission in our universe, might be stable in a universe with a much stronger nuclear force. If so, then they might provide a sufficiently fast reaction pathway to form heavier nuclei during the leptonic period. The usual route to lithium-7 formation, via beryllium-7, i.e.,

$$ {}^3_2\mathrm{He} + {}^4_2\mathrm{He} \to {}^7_4\mathrm{Be} \to {}^7_3\mathrm{Li} + \mathrm{e}^+ + \bar{\nu}_\mathrm{e} \tag{5} $$

will provide only the usual trace quantities of lithium-7. This is because the rate of reaction (5) is about 5 orders of magnitude slower than the rates of reactions (3) and (4). So the helium-3 disappears too quickly, via reaction (4), for reaction (5) to have a chance to produce any significant amounts of beryllium-7.

It is interesting to speculate whether this scenario might provide a successful outcome for Gamow's programme, i.e. to account for chemical elements via Big Bang nucleosynthesis. However, we need not be too concerned about it. For our present purposes it suffices that the *most* hydrogen that can remain is as given in the Table of Section 7. If nuclei beyond helium-4 are formed, then even these protons might be consumed in the process.

9. What Is The Upper Bound For g_s If The Weak Force Is Weaker?

$\tilde{G}_F$ **factored by…**	**Leptonic Reactions Freeze Out at Time…**
0.01	~0.001 sec
0.05	~0.01 sec
0.25	~0.1 sec

Thus, if we adopted ~3% hydrogen as the minimum that could result in life (for sake of argument – who knows) then the upper bound on g_s would be 4.3 times its value in this universe. However, if the Fermi constant, G_F, were less than 1% of its actual value then there would be even less than 3% hydrogen irrespective of the magnitude of the strong coupling constant, since the leptonic reactions would have frozen out earlier than 1 millisecond anyway.