Chapter 9 – The Structure and Stability of Matter: The Stability of Nuclei

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1. Introduction

In Chapter 1 the composition of atoms was described in outline. To be capable of forming interesting structures these atoms must have at least two properties: they must be stable and they must be able to combine together.

The requirement for atoms to be able to bind together in some sense is unavoidable if we wish to achieve complex structures. It is natural to envisage this as synonymous with solid matter. Is it possible to imagine an organism composed of a number of gas clouds bound by their mutual gravity? If gravity were the only inter-atomic force in operation, the entire metabolism and 'biochemistry' of the organism would need to be made coherent through gravitational bonding. This would require the organism to be on a colossal scale. It is difficult to see how gravitational binding would preserve any genetic code with sufficient fidelity to allow the Darwinian process to operate successfully and hence to permit biological evolution.

The requirement for stability needs little defence. One can, perhaps, imagine a living organism composed of unstable atoms. Since organisms continually renew their own living cells, it is possible to envisage the organism keeping one step ahead of destruction. They would need to pay special attention to the integrity of their genetic code. An extremely high degree of fidelity in passing on the genetic code to descendants is crucial for the Darwinian process of evolution to be successful. However, the environment of such an organism must provide a source of atoms for this constant renewal. If the atoms are intrinsically unstable, the environment must be continually manufacturing replacements. But if the environment were awash with energy compatible with the requirements for atomic synthesis then it is hard to believe it would not destroy the organism. Implicitly we are appealing to a hierarchy of energy scales: the larger the spatial scale the smaller the bond energy. Thus, the bond energy hierarchy is,

nuclear >> atomic > molecular > inter-molecular > configurational

Nuclear binding energies are up to ~8.8 MeV per nucleon. Atomic electron binding energies vary from a few eV, for the weakly held valance electrons in alkali metals, to the order of 100 keV for the innermost electrons of the heavier elements. The inter-atomic bonds comprising molecules have energies of a few eV. For example, the ionic bond of crystalline sodium chloride is ~2eV, whereas the particularly strong covalent OH bond in water is ~4.8eV. Inter-molecular interactions are barely bonds at all. Examples include van der Waals forces between polarisable molecules, and the hydrogen bond which is crucial to the peculiar properties of water. The energies of these are roughly 0.02eV and ~0.1eV respectively.

Finally, by the 'configurational' energy we mean the energy required to distort the shape of a large molecule without actually breaking any of its bonds. The biological properties of large molecules are determined for the most part by their geometry. The analogy with Lego bricks is particularly accurate in this case. Consequently, if a molecule is twisted into a new shape, but without breaking any of its bonds, it may be

rendered biologically ineffective. The denaturing of proteins by heating is an example. Thus, the energy required to rotate a protein bond, around 0.03eV, when equated with kT implies a 'cooking' temperature of approaching 100^{o}C (think of boiling an egg).

In this and the next few Chapters we shall mostly be concerned with the constraints on the universal constants arising from the requirement that matter be stable. Stability must be ensured at a number of different levels,

- Molecules must be stable against both dissociation and against collapsing into 'big atoms'. This must be true for a wide range of molecules, including very complex molecules capable of supporting biochemistry. These large molecules must be very stable indeed since they are the medium for the genetic code. A very high degree of fidelity of the genetic code down the generations is essential for Darwinian evolution to work.
- Atomic electrons must have stable orbits; they must be stable against dissociation and against being captured by the nucleus. The key atoms comprising biologically crucial molecules must, obviously, be as stable as the molecules, and for the same reason. There must be a sufficient diversity of different stable atoms to provide a rich biochemistry.
- A sufficiently diverse range of nuclei must be both bound and stable against radioactive disintegration to give rise to the biologically required spectrum of elements.

In this Chapter we shall consider the stability of nuclei. Chapter 10 will consider the stability of atoms, and Chapter 11 the stability of molecules. In Chapter 12 we will consider briefly the strength of solids, and how this constrains the possible size of planets (as opposed from asteroids) and also the size of planetary life forms.

2. The Binding and Stability of Nuclei

2.1 The Liquid Drop Model

Nuclei are bound by the strong nuclear force. Unfortunately a detailed quantitative theory which predicts the binding energies for every nucleus does not exist. Instead there are various approximate models. The model that is most useful for discussing the stability of nuclei is the liquid drop model. This is a semi-empirical model in which an algebraic form for the binding energy is motivated by theoretical considerations, but the coefficients of the terms in the expression are found by fitting to experimental data.

The most important feature of the nucleon-nucleon interaction is that it saturates. This means that a given nucleon in a large nucleus does not interact simultaneously with all the other nucleons, but only with a small number of them. It can be envisaged as interactions between nearest neighbours only, as for molecules in a liquid. In this respect the nucleon-nucleon force is similar to the chemical bonds between atoms in a large molecule. This behaviour contrasts with, say, the electrostatic interaction. In a bunch of Z equal charges, q, the electrostatic energy is proportion to Z^2 (see below). In contrast, for a force which saturates, the associated binding energy for a bunch of A such particles increases proportionally to A for large A. Thus, the dominant term in the liquid drop model for the nuclear binding energy is $a_v A$ where a_v is some coefficient to be determined, and A is, as usual, the total number of nucleons (i.e. the

atomic mass). The subscript 'v' denotes 'volume' since this dominant energy can be regarded as proportional to the volume of the nucleus (i.e. A $\propto$ volume).

The analogy with a liquid drop comes into play by recalling that such a droplet exhibits surface tension. Surface tension is caused by the forces acting on a surface molecule due to the molecules within the liquid not being balanced by any molecules on the other side of the surface. Thus, surface tension represents a deficit in binding energy for surface molecules (or nucleons) due to the absence of molecules (or nucleons) on the outside. The liquid drop model therefore includes a negative contribution to the binding energy which is proportional to the surface area, i.e. $a_s A^{2/3}$.

The other obvious negative contribution to the binding energy of a nucleus is the Coulomb energy. It is a positive potential energy, and hence a negative contribution to the binding energy. The Coulomb repulsion tends to make the nucleus fly apart. The Coulomb energy associated with a charge Q spread uniformly throughout a sphere of radius R is proportional to Q^2/R (to be precise, it is $\frac{3}{5}\cdot\frac{Q^2}{4\pi\varepsilon_0 R}$ in MKSA units). The total charge is just Ze, where Z is the number of protons (the atomic number), and the radius is $R \propto A^{1/3}$. Thus, the Coulomb term is a negative contribution to the nuclear binding energy which can be written $a_c Z^2 / A^{1/3}$. In this case, the coefficient a_c is known so long as we know the nuclear size for some A. For this we use r_0 = 1.22 fm for A = 1. We thus find $a_c = \frac{3}{5}\cdot\frac{\alpha\hbar c}{r_0} = 0.71$ MeV.

So far there is nothing in our formulation of the binding energy to prevent a nucleus consisting entirely of neutrons. In fact, nuclei are stable only if the numbers of neutrons (N) and protons (Z) are comparable. The reason can be understood qualitatively from a shell model approach. The effect of the exclusion principle is that all the nucleons can partake of the maximum binding energy (e.g. as represented by the algebraic sum of the volume and surface terms, above) only if their numbers are equal, N = Z. If not, an excess of $|N - Z|$ nucleons are pushed up to higher energy levels by the exclusion principle. Thus, there is a reduction in the binding energy if N and Z are not equal. This is true whether the excess consists of neutrons or protons. Hence, the change in the binding energy due to the excess is negative regardless of the sign of N – Z. This suggests the binding energy contribution is proportional to $-(N - Z)^2$. However, we expect the binding energy adjustment for a given $|N - Z|$ to be larger in a smaller nucleus, so we normalise the squared expression by the total number of nucleons, i.e. the contribution is $-a_a (N - Z)^2 / A$. This is known as the 'asymmetry energy'. The argument is perhaps less than convincing as regards the precise algebraic form. Ultimately its justification is empirical. In particular, the dependence on $-(N - Z)^2$ is strongly supported experimentally.

In truth, the binding energy of a nucleus is not as smooth a function of N and Z as is implied by the above simple considerations. For one thing, the shell model suggests

that nuclei should be particularly strongly bound if they correspond to full shells (in analogy with the noble gases). This is born out experimentally when N or Z is one of the 'magic numbers' 20, 28, 40, 50, 82 or 126. We will not attempt to include this refinement into the liquid drop model. However, another systematic feature of nuclear binding energies is that nuclei with both even-N and even-Z tend to be particularly stable, i.e. have larger binding energies, and the opposite is true for nuclei with both odd-N and odd-Z. This is due to 'pairing energy' and is incorporated into the liquid drop model as a term $\delta = a_p / A^{3/4}$, the form of this being purely empirical. Thus, the binding energy is increased by δ for even-N/even-Z nuclei, but decreased by δ for odd-N/odd-Z nuclei. No change is made for nuclei with odd A.

Adding all the terms together, the liquid drop model estimates nuclear binding energies as,

$$B(Z,A) = a_v A - a_s A^{2/3} - a_a \frac{(A-2Z)^2}{A} - S_{NZ} \frac{a_p}{A^{3/4}} - a_c \frac{Z^2}{A^{1/3}} \qquad (2.1)$$

where, S_{NZ} is –1, +1 or 0 accordingly as N and Z are even/even, odd/odd or of different parity. Different authors have offered different empirical fits for the coefficients in (2.1). Here we shall use: $a_v = 15.75$, $a_s = 17.8$, $a_a = 23.7$ and $a_p = 33.5$ (all in MeV), together with $a_c = 0.71$ MeV, as derived above.

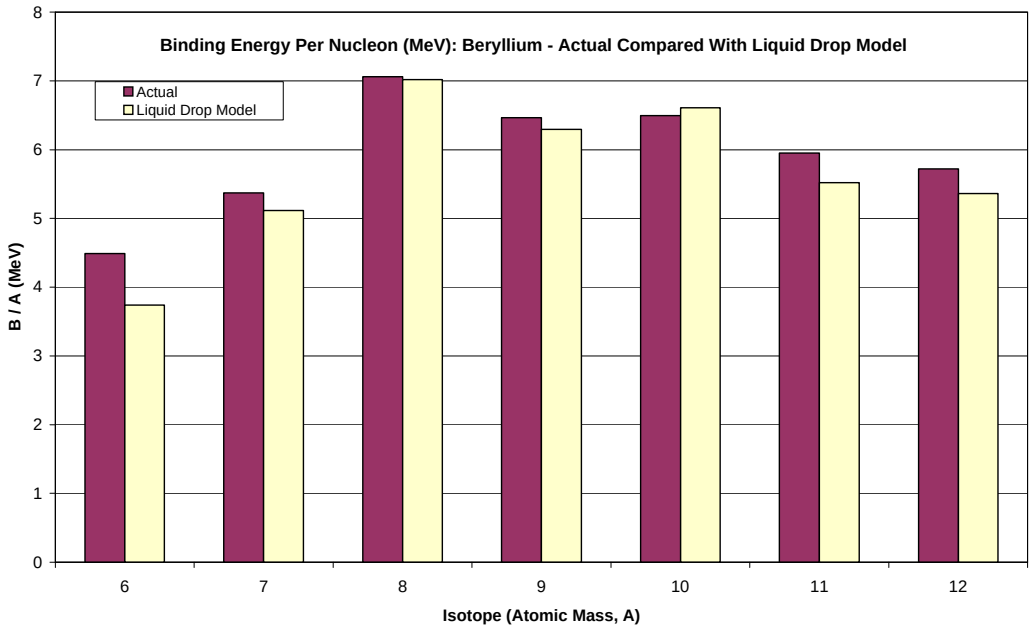
What we wish to do is to use some modification of (2.1) to explore how the stability of nuclei is affected by changes in the strength of the nuclear force and/or the electrostatic force. Before doing so we first examine how well Equ.(2.1) performs in *this* universe.

2.2 Liquid Drop Model Compared With Measured Nuclear Binding Energies

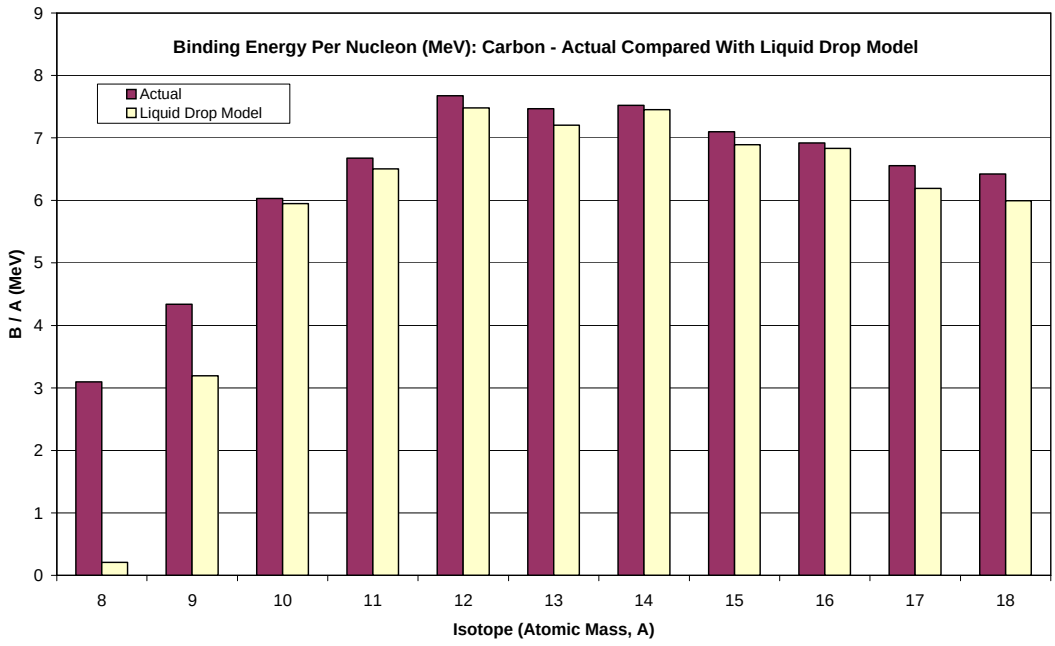
The liquid drop model was not devised to perform well for small values of N or Z. Nevertheless, the isotopes of helium, lithium and beryllium with the largest binding energies per nucleon (B/A) show reasonable agreement,

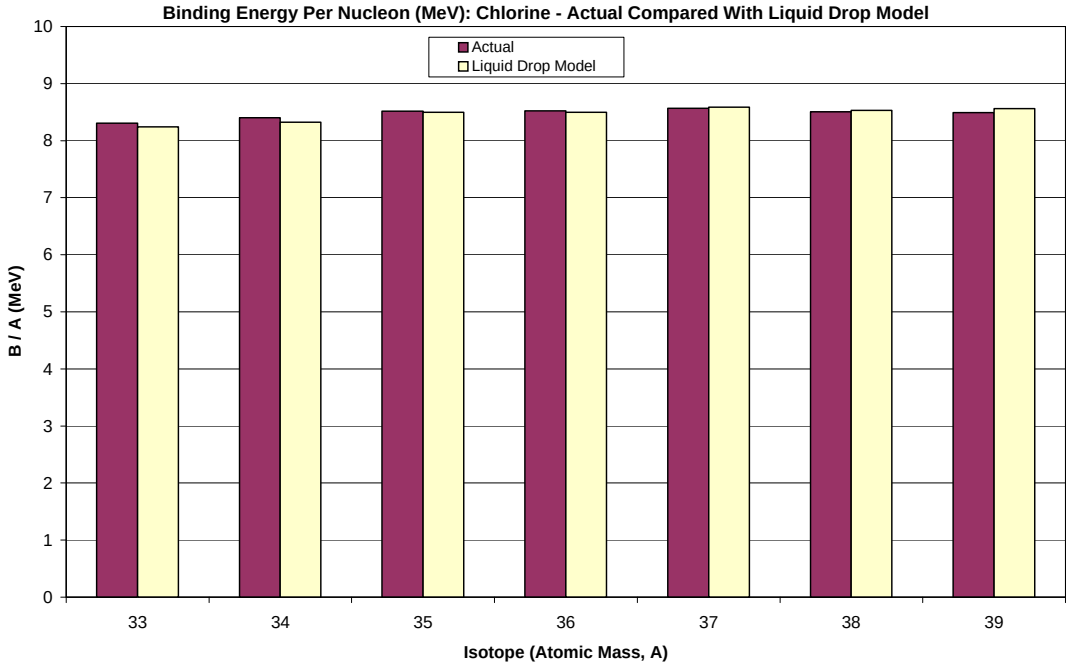
Isotope	Actual B/A (MeV)	B/A, Equ.(2.1) (MeV)
$^4_2$He	7.074	7.051
$^7_3$Li	5.606	5.484
$^8_4$Be	7.062	7.020

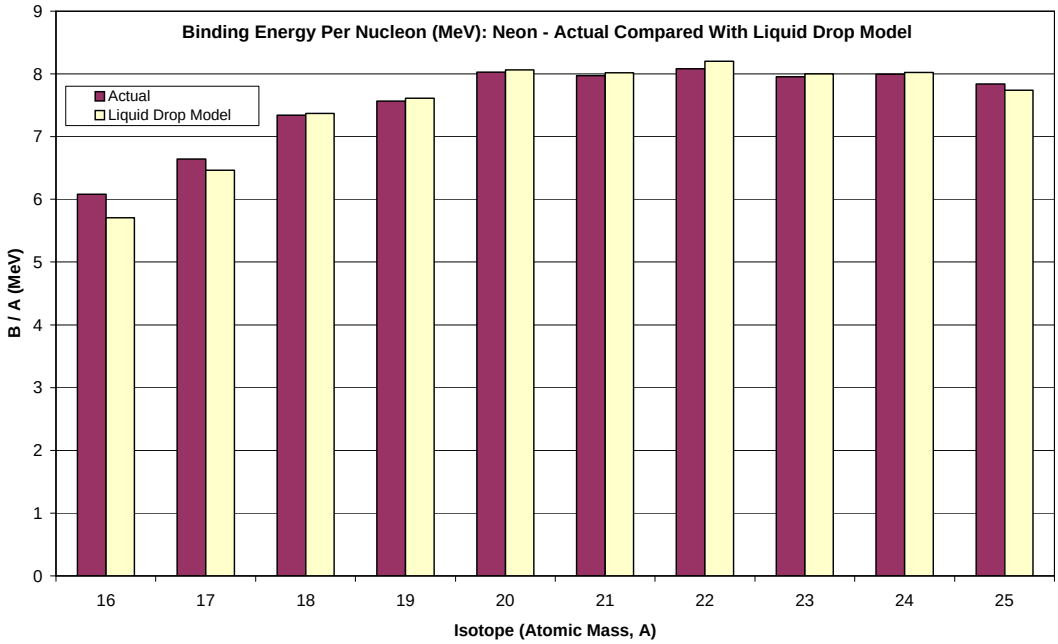
[Note that $^8_4$Be is unstable against decay into two alpha particles, and the liquid drop binding energies would 'predict' this]. For beryllium, the comparison between the liquid drop model and the actual binding energies is shown below for a range of different isotopes:-

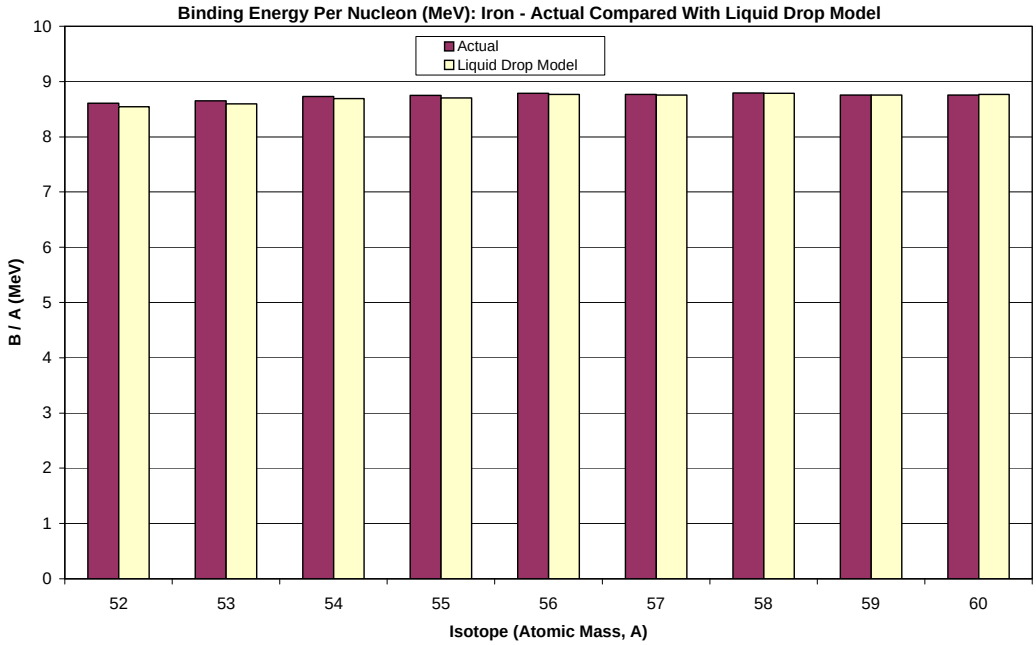


The agreement is reasonable given that this is a light nucleus for which the liquid drop model is not ideal. Similar plots follow for carbon, chlorine, neon and iron:-

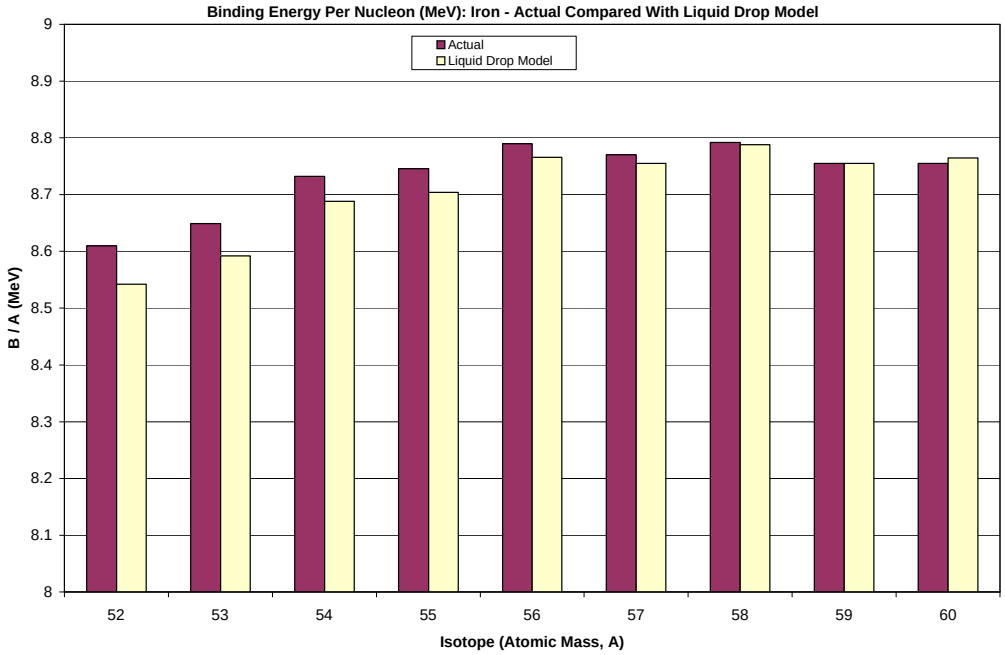



Binding Energy Per Nucleon (MeV): Chlorine - Actual Compared With Liquid Drop Model
Actual
Liquid Drop Model
B / A (MeV)
0
1
2
3
4
5
6
7
8
9
10
33
34
35
36
37
38
39
Isotope (Atomic Mass, A)

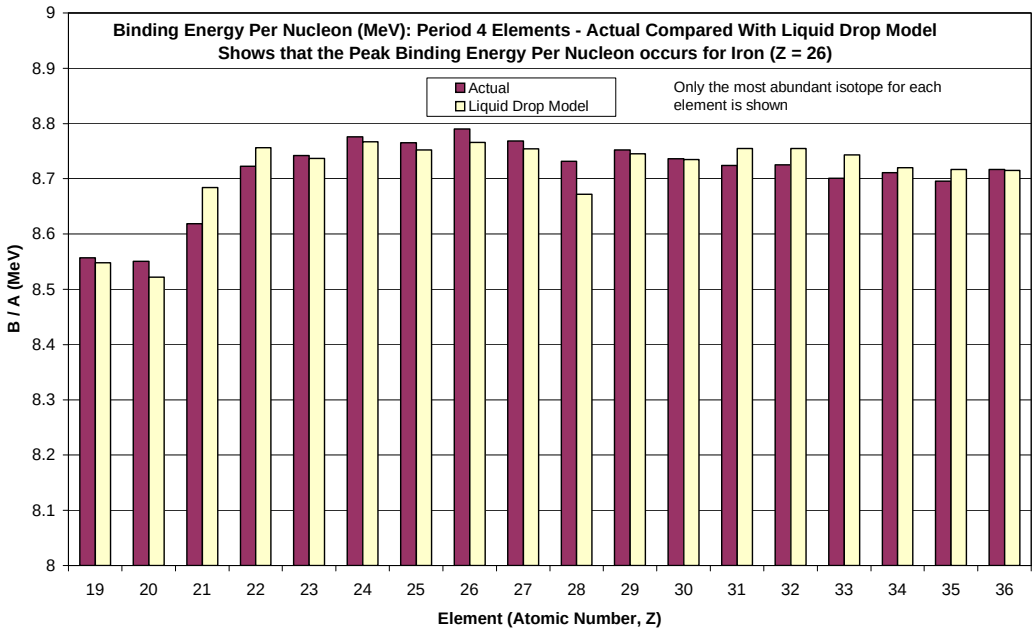

Binding Energy Per Nucleon (MeV): Neon - Actual Compared With Liquid Drop Model
Actual
Liquid Drop Model
B / A (MeV)
0
1
2
3
4
5
6
7
8
9
16
17
18
19
20
21
22
23
24
25
Isotope (Atomic Mass, A)



Agreement becomes better for the heavier nuclei, but for these nuclei there is very little variation in binding energy when A is varied by +/-4 or so from the most abundant stable isotope. The case of iron is shown on an expanded energy scale below,

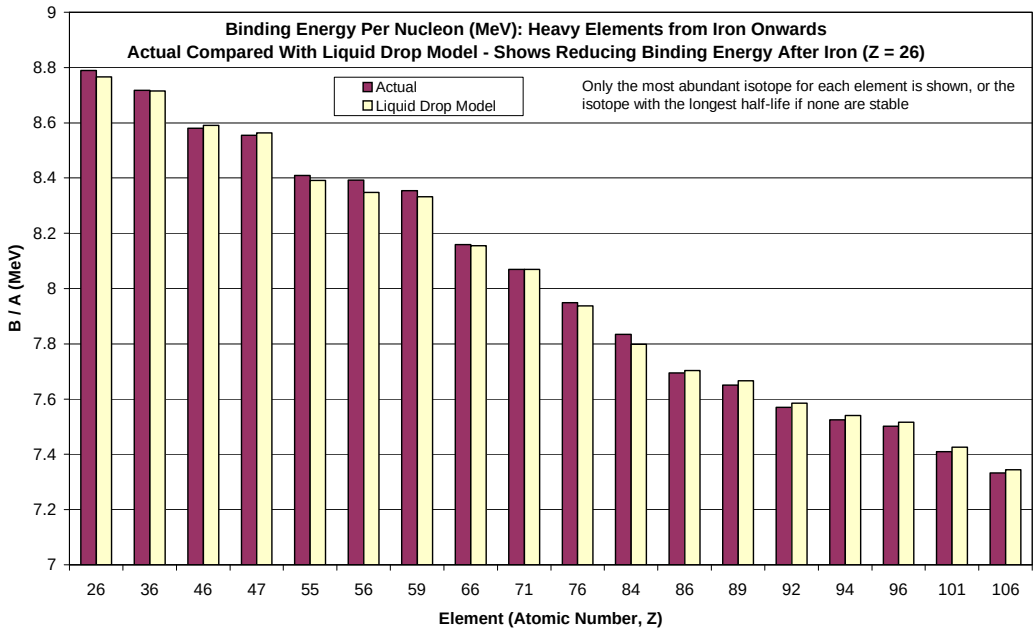


Iron isotope $^{58}_{26}\mathrm{Fe}$ has the largest binding energy per nucleon of any nuclide (the actual binding energy is 8.792 Mev; the liquid drop model, Equ.(2.1), gives 8.788 MeV). Strangely this isotope accounts for only 0.28% of terrestrial iron[1]. $^{56}_{26}\mathrm{Fe}$, which accounts for 91.75% of terrestrial iron, has very nearly as large a binding energy, however, at 8.790 MeV/nucleon (cf. 8.766 MeV from Equ.2.1). In fact, Iron-56 has the largest binding energy as regards the most abundant isotopes for each element, as illustrated below for Period 4 of the periodic table (NB: $Z_{iron} = 26$):-

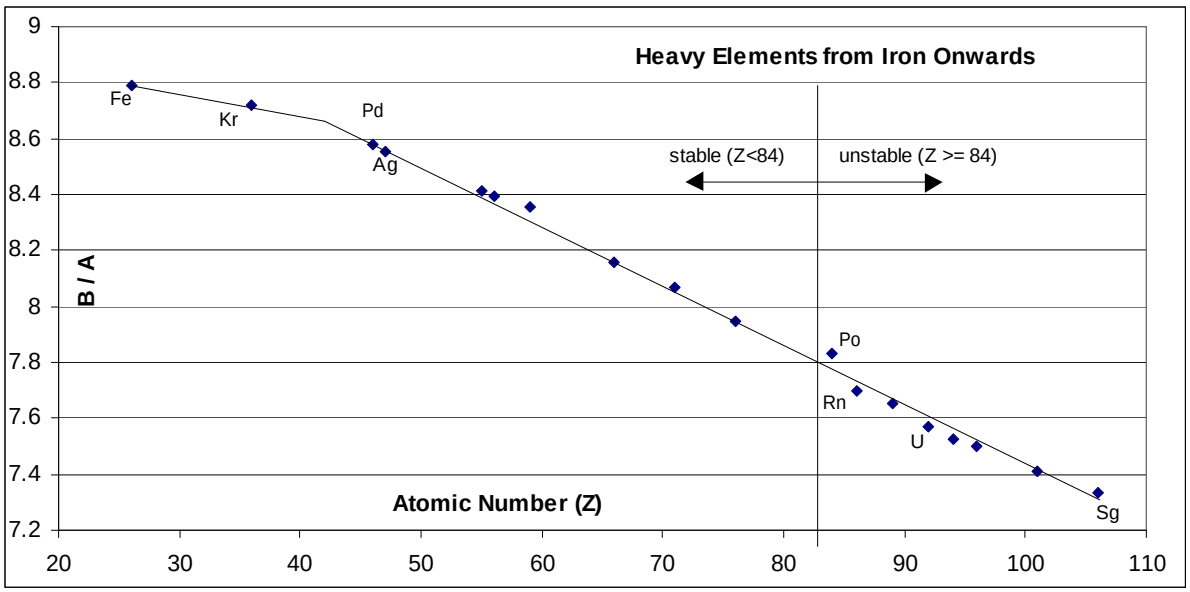


The trend of reducing binding energy per nucleon beyond iron continues to the end of the periodic table, as illustrated below for a randomly chosen sample of heavier elements (note that the x-axis is not linear in Z):-

[1] It is stable, so why does isotope $^{56}_{26}\mathrm{Fe}$ account for 91.75% of terrestrial iron whereas $^{58}_{26}\mathrm{Fe}$ accounts for only 0.28%? I'm guessing, but it is probably because 56 is divisible by 4 and 58 is not. What's more, half of 56 is also divisible by 4. Think of how the elements are synthesised in stars, largely from helium-4.



The same actual binding energy data is shown on a graph against linear-Z below:-

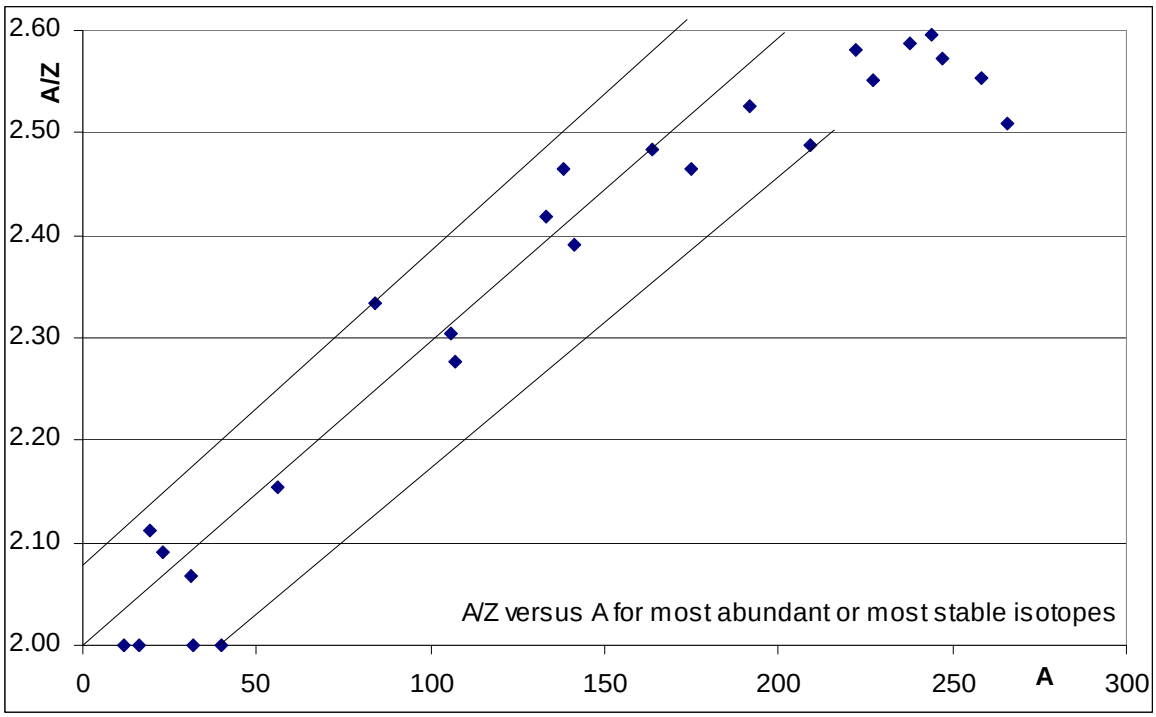


Which elements are radioactive? Of course *any* element, that is any Z, has unstable isotopes. One need only consider an isotope with an adverse number of neutrons, either too many or too few. However, there is a fairly clear dividing line between those elements which have at least one stable isotope and those elements which have none. This occurs between bismuth (Z = 83) and polonium (Z = 84), as shown on the above graph. From polonium onwards there are no stable isotopes. Up as far as bismuth almost all elements have at least one stable isotope, the only exceptions being technetium (Tc, Z = 43) and promethium (Pm, Z = 61).

The unstable elements from polonium onwards have a huge range of half-lives, from milliseconds to 24 times the age of the universe in the case of protactinium-231 (Pa, Z = 91). Uranium-238 and thorium-232 also have half-lives in the billions of years. It is tempting to regard such isotopes as being virtually stable. This is true in the sense that a negligible proportion of a given sample will decay over, say, a mere million years. Nevertheless, a gram of uranium-238 undergoes 17,900 atomic disintegrations per second. The moral is that, whilst there are a lot of seconds in a billion years, this number is tiny compared with the number of atoms in a gram of material. The half-lives of the longest lived isotopes of each element are listed below (elements not listed being infinite),

Z	Element	A	Half-life
43	Tc	97 or 98	4.2 million years
61	Pm	145	17.7 years
84	Po	209	102 years
85	At	210	8 hours
86	Rn	222	3.8 days
87	Fr	223	22 mins
88	Ra	226	1600 years
89	Ac	227	21.7 years
90	Th	232	14 billion years
91	Pa	231	328 billion years
92	U	238	4.47 billion years
93	Np	237	2 million years
94	Pu	244	80 million years
95	Am	243	7370 years
96	Cm	247	15.6 years
97	Bk	247	1380 years
98	Cf	251	898 years
99	Es	252	472 days
100	Fm	257	100 days
101	Md	258	52 days
102	No	259	58 mins
103	Lr	262	4 hours
104	Rf	261	65 secs
105	Db	262	35 secs
106	Sg	266	26 secs
107	Bh	264	0.4 secs
108	Hs	269	10 secs
109	Mt	268	21 millisec
110	Ds	271	69 millisec
111	Rg	272	4 millisec
112		285	29 secs
113		287	20 minutes
114		289	2.6 secs
115		291	1 min
116		293	61 millisec
117		292	50 millisec
118		294	1.8 millisec

For the heavier elements, the stable or longest lived isotopes have a larger number of neutrons than protons. This is because, by increasing N above Z, the dominant 'volume' contribution to the binding energy in Equ.(2.1) can be increased without increasing the Coulomb term (which reduces the binding energy). However, there is a limit to this because, if N differs too much from Z the 'asymmetry energy' term in Equ.(2.1) will reduce the binding energy. Hence, there is some N > Z which maximises the binding energy per nucleon, as seen in the preceding plots. Considering only the most abundant or longest lived isotopes, the A/Z ratio is ~2.0 for calcium (Z = 20), increasing to ~2.15 for iron (Z = 26), and to ~2.3 for krypton (Z = 36). By the instability threshold at polonium (Z = 84) A/Z has reached ~2.5. Thereafter it remains between ~2.5 and ~2.6 for the heavier, unstable, elements. The ratio A/Z for the most abundant or the longest lived isotopes for a given element are illustrated for a random selection of elements in the graph below.



2.3 Theory of Nuclear Stability (This Universe)

How can we use the estimate of an isotope's binding energy, as given by Equ.(2.1), to determine whether it is stable? We distinguish five types of nuclear instability,

- Instability against breaking into two smaller nuclei of roughly equal size;
- Instability against emission of a smaller fragment, specifically an alpha particle;
- Instability with respect to deformation from a spherical shape;
- Instability against beta decay (electron emission);
- Instability against inverse beta decay (positron emission);
- Instability against double beta decay or double positron emission;
- Instability against electron capture.

These are examined in turn in the following sub-sections.

2.3.1 Instability Against Symmetrical Fission

The fission of a nucleus (Z, A) into two nuclei $(Z_1, A_1) + (Z_2, A_2)$, where $Z_1 + Z_2 = Z$ and $A_1 + A_2 = A$, is energetically possible provided that,

$$B(Z_1, A_1) + B(Z_2, A_2) > B(Z, A) \qquad (2.2)$$

Symmetrical fission is when $Z_1 \approx Z_2$ and $A_1 \approx A_2$, i.e. the two daughter nuclei are almost the same size. For perfectly symmetrical fission, condition (2.2) becomes,

$$2B(Z/2, A/2) > B(Z, A) \qquad (2.3)$$

This condition is likely to be fulfilled for $Z \geq 52$, since the binding energy per nucleon is greatest for iron (Z = 26), and the preceding graph shows that there is a reasonably continual reduction in B/A with increasing Z beyond Z = 26. However, this is not entirely clear because the stable isotopes with Z/2 protons will tend to have fewer than A/2 nucleons. Evaluation shows that symmetrical fission of the most stable isotope is energetically possible for Z = 41 and for $Z \geq 43$. Specifically we find, from Equ.(2.1),

(2.4)

$^{44}_{19}\mathrm{K} + {}^{45}_{20}\mathrm{Ca} < {}^{89}_{39}\mathrm{Y}$ (stable)

$2\times {}^{45}_{20}\mathrm{Ca} < {}^{90}_{40}\mathrm{Zr}$ (stable)

$^{46}_{20}\mathrm{Ca} + {}^{47}_{21}\mathrm{Sc} > {}^{93}_{41}\mathrm{Nb}$ (fission energetically possible)

$2\times {}^{48}_{21}\mathrm{Sc} < {}^{96}_{42}\mathrm{Mo}$ (stable)

$^{49}_{21}\mathrm{Sc} + {}^{50}_{22}\mathrm{Ti} > {}^{99}_{43}\mathrm{Tc}$ (fission energetically possible)

$2\times {}^{50}_{22}\mathrm{Ti} > {}^{100}_{44}\mathrm{Ru}$ (fission energetically possible)

$^{51}_{22}\mathrm{Ti} + {}^{52}_{23}\mathrm{V} > {}^{103}_{45}\mathrm{Rh}$ (fission energetically possible)

where the symbols represent the binding energies, and the isotopes on the RHS are those with the greatest binding energy for the given element. Thus it is energetically possible for niobium (Z = 41) and elements from technetium onwards ($Z \geq 43$) to undergo symmetrical fission. With the exception of molybdenum, the most stable isotopes of all elements with $Z^2/A > 18$ are energetically susceptible to symmetrical fission.

Notwithstanding this, elements with Z between 41 and 83 *do* display stable isotopes (with the exception of Z = 43 and Z = 61). Conditions (2.2), (2.3) or (2.4) are necessary but not sufficient conditions for instability against symmetrical fission. A sufficient condition for instability is given in Section 2.3.3, below.

2.3.2 Instability Against Alpha Decay

The particular case of instability to alpha particle emission can also be addressed using Equ.(2.2) which becomes,

$$B(Z-2, A-4) + B(2, 4) > B(Z, A) \qquad (2.5)$$

where B(2, 4) is the binding energy of the helium-4 nucleus (28.296 MeV). Elements up to and including iron will be immune to alpha decay (that is, their isotopes with

maximum B/A will be immune) because their binding energy per nucleon will be greater than that of the potential daughter nucleus, as well as greater than that of an alpha particle. (One exception to this is the unstable $^{8}_{4}\mathrm{Be}$ which decays into two alpha particles).

For elements beyond iron we can approximate the binding energy for the most abundant isotope, or that with greatest B/A, as,

$$B(Z) = [9.560 - 0.0213Z].A \qquad (2.6)$$

This follows from the above graphs and is a good approximation for Z > 43 or so. Substitution of (2.6) into (2.5) yields,

$$Z + A/2 > 118.7 \qquad (2.7)$$

as the (necessary) condition for instability to alpha decay. Assuming A/Z = 2.3, as for the 'middle weight' elements around palladium, this implies alpha emission may occur for Z > 55. If A/Z = 2.6 is used, which is the greatest value this ratio takes, we find Z > 51 is the condition for alpha emission. Again these are necessary but not sufficient conditions for alpha decay. Thus, alpha decay *cannot* occur for the isotopes of elements with Z < 51 which have the largest B/A. This is correct, of course, since all such isotopes are stable (apart from technetium, but this cannot alpha-decay either: $^{98}_{43}\mathrm{Tc}$ and $^{99}_{43}\mathrm{Tc}$ undergo beta decay, whereas $^{97}_{43}\mathrm{Tc}$ undergoes inverse beta decay, emitting a positron).

2.3.3 Instability Against Shape Change

In sub-sections 2.3.1 and 2.3.2 we have found necessary conditions for fission of the most stable isotope of each element, namely (roughly) Z > 42 for symmetrical fission and Z > 51 for alpha emission. However, these are not sufficient conditions for instability of the nuclei, they are merely necessary to ensure that energy is conserved in the postulated decays. Consequently it is not surprising that heavier elements are stable in practice. General fissile instability sets in only from polonium onwards ($Z \geq 84$, with the exceptions of technetium and promethium). In this sub-section we derive a sufficient condition for nuclear instability against symmetrical fission. This is derived by considering the energy change which occurs if the nucleus distorts slightly from its assumed spherical initial geometry. If the binding energy increases when the nucleus distorts, it follows that it must be unstable with respect to such distortions. Thus, a sufficient condition for instability consists of:-

(a)A series of infinitesimal distortions which lead continuously from the initial spherical shape to a shape approximating two spheres each of half the volume on the point of separating, together with,

(b)A binding energy which increases monotonically as the deformation proceeds.

However, for simplicity we shall assume that the later stages of such a distortion will be unstable if the earlier stages are unstable. Thus, it is claimed to be sufficient to consider the initial sphere distorting into an ellipsoid of small eccentricity. This is a long established method which goes back to Bohr and Wheeler (1939).

We shall consider here only prolate ellipsoids (i.e. a stretched sphere). If this is found to be unstable then it provides a valid sufficient condition for instability. However it would be of interest also to consider an oblate ellipsoid (i.e. a flattened sphere) since it is not obvious that this does not provide a more onerous stability condition. However, an oblate spheroid seems less likely to break into two parts than, perhaps, three or four parts.

The distortion to a prolate ellipsoid is assumed to occur at constant volume. Hence, the dominant volume term in the liquid drop model, Equ.(2.1), is unchanged. Similarly, the asymmetry and pairing energies are unchanged in Equ.(2.1). This leaves just the surface term and the Coulomb energy which change on distortion:-

$$\Delta B(Z,A) = \Delta\left\{-a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}\right\} \qquad (2.3.3.1)$$

Since a sphere is the shape with the smallest surface area for a given volume, it follows that the surface area must increase on distortion. The surface energy term therefore increases in magnitude. Since the surface energy reduces the binding energy, it follows that the surface term causes a *reduction* in the binding energy upon distortion. Consequently, if it were not for the Coulomb energy, nuclei would always be stable against small distortions into a prolate elliptical shape. (This is consistent with the behaviour of liquid droplets, for which a spherical shape is the stable configuration due to surface tension – in the absence of gravity, that is).

However, since the distortion leads to a greater 'spreading out' of the charge, it is to be expected that the Coulomb energy will reduce in magnitude. Detailed calculation confirms this. Since the Coulomb energy also reduces the binding energy, it follows that the Coulomb term causes an *increase in* binding energy upon distortion. Thus, if it were not for the surface term acting in opposition, nuclei would always be unstable against small distortions, i.e. the Coulomb repulsion would always lead to fission if this were energetically possible (in accord with Sections 2.3.1 and 2.3.2). Thus, whether a nucleus is stable or unstable against small distortions depends upon which is larger: the change in the Coulomb energy or the change in the surface energy.

In Appendix B1 we derive the changes in the surface energy and the Coulomb energy due to an assumed distortion into a prolate ellipsoid with semi-major axis c and semi-minor axis b given by,

$$c = R(1+\varepsilon) \quad \text{and} \quad b = R\left(1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2\right) \qquad (2.3.3.2)$$

where R is the radius of the original sphere, $\varepsilon << 1$ and the volume is constant to order ε^3. The ellipsoid is given by,

$$x = c \cdot \cos\theta \quad \text{and} \quad \sqrt{y^2 + z^2} = b \cdot \sin\theta \qquad (2.3.3.3)$$

where, $b = \sqrt{c^2 - a^2}$ and $a = c\sqrt{3\varepsilon}$ (2.3.3.4)

The surface area of the prolate ellipsoid is found to be,

$$S(\text{prolate}) = 4\pi R^2\left(1 + \frac{2}{5}\varepsilon^2 + O(\varepsilon^3)\right) \quad (2.3.3.5)$$

The Coulomb energy of the prolate ellipsoid, assuming the charge density within it remains uniform, is, to lowest order,

$$V_c = a_c \frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{5}\varepsilon^2 + O(\varepsilon^3)\right] \quad (2.3.3.6)$$

NB: In Appendix B1 we use a rather 'sledge hammer' method which derives the value of the numerical factor in the above expression to be –0.198, i.e. not exactly 1/5. However, Evans, and Barrow and Tipler, simply state 1/5. There may be a better derivation than mine.

Using (2.3.3.5) and (2.3.3.6) in (2.3.3.1) gives the change in binding energy upon distortion to be,

$$\Delta B(Z, A) = -a_s A^{2/3}\left[\frac{2}{5}\varepsilon^2\right] - a_c \frac{Z^2}{A^{1/3}}\left[-\frac{1}{5}\varepsilon^2\right] \quad (2.3.3.7)$$

This is positive, indicating instability, if Z is sufficiently large. Thus, our sufficient condition for instability is simply,

$$\frac{Z^2}{A} > 2\frac{a_s}{a_c} = 50 \quad (2.3.3.8)$$

For the heaviest elements we have A/Z ~ 2.5 so the instability condition becomes Z > 125. Thus we could not have more than ~125 stable elements given that the strong nuclear and electromagnetic forces take the strengths that they do.

Since (2.3.3.8) is a sufficient, but not a necessary, condition for instability, it is not surprising that lighter elements are actually unstable (namely all those from polonium, Z = 84, downwards). Thus, whilst Z > 42 is necessary for symmetrical fission to occur, instability is not certain until Z > 125. We may make a crude guess at a more accurate stability criterion by taking the simple average of the necessary and sufficient conditions. This suggests that the instability condition will be Z > (42 + 125) / 2 = 83 – which is actually bang-on!

In passing we note that the reason why a continuous deformation which monotonically increases the binding energy is not a *necessary* condition for instability is due to the possibility of quantum tunnelling. If nuclei behaved classically, then this criterion would be necessary, and nuclei could only be unstable for Z > 125. But quantum tunnelling allows fission to occur even if there is an intermediate state of

deformation which decreases the binding energy. Such states are 'jumped over' by quantum tunnelling.

2.3.4 Instability Against Beta Decay or Inverse Beta Decay

Beta decays may be written,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e + \overline{\nu}_e \tag{2.8} $$

The conservation of mass-energy, and the fact that the kinetic energy of the particles on the right cannot be negative, requires,

$$ ZM_p + NM_n - B(Z,A) > (Z+1)M_p + (N-1)M_n + m_e - B(Z+1,A) \tag{2.9} $$

which, inserting the values $M_p - M_n = 1.292$ MeV and $m_e = 0.511$ MeV, reduces to,

$$ B(Z,A) - B(Z+1,A) < M_n - M_p - m_e = 0.782\,\text{MeV} \tag{2.10} $$

Thus, beta decay is energetically possible if it leads to a daughter nucleus with a greater binding energy. However, it is also energetically possible if the binding energy decreases, so long as this decrease is less than the mass deficit.

Similarly, inverse beta decays (positron emission) can be written,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+} + \nu_e \tag{2.11} $$

The conservation of mass-energy, and the fact that the kinetic energy of the particles on the right cannot be negative, requires,

$$ ZM_p + NM_n - B(Z,A) > (Z-1)M_p + (N+1)M_n + m_e - B(Z-1,A) \tag{2.12} $$

which, inserting the values $M_p - M_n = 1.292$ MeV and $m_e = 0.511$ MeV, reduces to,

$$ B(Z-1,A) - B(Z,A) > M_n - M_p + m_e = 1.804\,\text{MeV} \tag{2.13} $$

Thus, inverse beta decay (positron emission) can only occur if the daughter nucleus has greater binding energy, and by an amount which exceeds the mass deficit.

[Confusion can arise because some sources quote the threshold for positron emission to be $2m_e = 1.022$MeV. This is the lower bound required difference in the *atomic* masses, including the full complement of electrons to make the atoms neutral. The daughter atom must eject an electron to become neutral (possibly by annihilation with the positron). The nucleon mass difference cancels from the atomic mass difference, and the ejected, or annihilated, electron makes the atomic mass difference $2m_e$. In short, the atomic mass difference is $2m_e$ because both a positron and an electron are emitted. Strictly, the bound is that the atomic mass difference should be $> 2m_e - \Delta B_e$, where ΔB_e is the decrease in the total binding energy of all the electrons, due partly to the missing electron and partly to the reduction in the nuclear charge.]

Double beta decays are simply two beta decays occurring simultaneously, i.e.,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}Y + 2e + 2\overline{\nu}_e \qquad (2.14) $$

The conservation of mass-energy, and the fact that the kinetic energy of the particles on the right cannot be negative, requires,

$$ ZM_p + NM_n - B(Z,A) > (Z+2)M_p + (N-2)M_n + 2m_e - B(Z+2,A) \qquad (2.15) $$

which, inserting the values $M_p - M_n = 1.292$ MeV and $m_e = 0.511$ MeV, reduces to,

$$ B(Z,A) - B(Z+2,A) < 2(M_n - M_p - m_e) = 1.564\,\text{MeV} \qquad (2.16) $$

Thus, double beta decay tends to occur if (2.16) is obeyed, but the criterion for single beta decay, (2.10), is not met.

Similarly, double inverse beta decay (double positron emission) is,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-2}Y + 2e^{+} + 2\nu_e \qquad (2.17) $$

The conservation of mass-energy, and the fact that the kinetic energy of the particles on the right cannot be negative, requires,

$$ ZM_p + NM_n - B(Z,A) > (Z-2)M_p + (N+2)M_n + 2m_e - B(Z-2,A) \qquad (2.18) $$

which, inserting the values $M_p - M_n = 1.292$ MeV and $m_e = 0.511$ MeV, reduces to,

$$ B(Z-2,A) - B(Z,A) > 2(M_n - M_p + m_e) = 3.608\,\text{MeV} \qquad (2.19) $$

Thus, double positron emission tends to occur if (2.19) is obeyed, but the criterion for single positron emission, (2.13), is not met.

We can now use the liquid drop model, Equ.(2.1), to determine whether the inequalities of (2.10), (2.13), (2.16) or (2.19) are obeyed. Provided that Equ.(2.1) is a sufficiently accurate representation of the nuclear binding energies, beta decay, or inverse beta decay, cannot occur if these inequalities are not respected. We have examined 40 isotopes at random to examine the performance of Equ.(2.1) as a predictive tool for beta and inverse beta decays, as listed in the following Table. The black data indicate that the corresponding inequality is respected (predicting that decay is possible), whereas the data in red indicate that the inequality is not respected and hence the isotope should be stable. Equ.(2.1) correctly indicates the decay mode for the black and red data. Data in blue indicate where Equ.(2.1) gives an incorrect indication of stability or instability, the data given being derived from the true (measured) binding energies. The green data relate to double decay modes, for details of which see the footnotes.

Isotope	**Actual Decay Mode***	$B(Z,A) - B(Z+1,A)$ **(criterion: <0.782)**	$B(Z-1,A) - B(Z,A)$ **(criterion: >1.804)**
$^{13}_{8}O$	ε		19.114
$^{14}_{8}O$	ε		1.933
$^{15}_{8}O$	ε		4.319
$^{16}_{8}O$	stable	19.090	-10.072
$^{17}_{8}O$	stable	4.695	-7.013
$^{18}_{8}O$	stable	7.395	-19.404
$^{19}_{8}O$	β	-5.457	
$^{20}_{8}O$	β	-2.690	
$^{21}_{8}O$	β	-13.682	
$^{39}_{20}Ca$	ε		8.167
$^{40}_{20}Ca$	2ε		6.296(3)
$^{41}_{20}Ca$	ε		3.407
$^{42}_{20}Ca$	stable	10.177	-2.864
$^{43}_{20}Ca$	stable	3.896	-0.912
$^{44}_{20}Ca$	stable	5.702	-6.851
$^{45}_{20}Ca$	β	-0.243	
$^{46}_{20}Ca$	2β	0.156(2)	
$^{37}_{19}K$	ε		7.885
$^{38}_{19}K$	ε		9.698
$^{39}_{19}K$	stable	8.167	0.217(4)
$^{40}_{19}K$	β	-0.529(1)	
$^{41}_{19}K$	stable	3.407	-1.632
$^{42}_{19}K$	β	-2.864	
$^{43}_{19}K$	β	-0.912	
$^{44}_{19}K$	β	-6.851	
$^{53}_{25}Mn$	ε		2.104
$^{54}_{25}Mn$	ε		3.791
$^{55}_{25}Mn$	stable	2.629	-1.193
$^{56}_{25}Mn$	β	-2.268	
$^{57}_{25}Mn$	β	-0.570	
$^{58}_{25}Mn$	β	-5.272	
$^{98}_{43}Tc$	β	0.588	
$^{99}_{43}Tc$	β	0.489(1)	
$^{222}_{87}Fr$	β	-0.710	

$^{223}_{87}$Fr	β	0.089	
$^{227}_{89}$Ac	β	0.7718	
$^{231}_{90}$Th	β	0.439	
$^{237}_{92}$U	β	0.427	
$^{238}_{92}$U	α	1.190	
$^{239}_{92}$U	β	-0.239	

*β = beta decay; 2β= double beta decay,
ε = inverse beta decay; 2ε = double inverse beta decay

FOOTNOTES to the above Weak Decay Table
(1)Equ.(2.1) actually gave a result greater than 0.782MeV, thus falsely indicating beta decay to be impossible. This is due to the inaccuracy of Equ.(2.1) for these isotopes. The figure quoted is the true binding energy difference.
(2)For double beta decay the quantity given is B(Z,A) – B(Z+2,A). Note that this isotope is predicted to be stable against single beta decay since B(Z,A) – B(Z+1,A) = 1.615 > 0.782, but unstable against double beta decay. Thus, the predictions are faithful to the observed double beta decay mode.
(3)For double inverse beta decay the quantity given is B(Z-2,A) – B(Z,A). To be susceptible to double beta decay this must exceed 2 x 1.804 = 3.608, which it does. Also note that B(Z-1,A) – B(Z,A) is less than 1.804. Hence, the prediction is that the isotope is stable against inverse beta decay, but unstable against double inverse beta decay – thus correctly predicting the observed double inverse beta decay mode.
(4)Equ.(2.1) actually gave a result greater than 1.804MeV, thus falsely indicating inverse beta decay to be possible. This is due to the inaccuracy of Equ.(2.1) for these isotopes. The figure quoted is the true binding energy difference.

We see that the predictions of the model are generally good (i.e. black data in the correct decay mode, and red data where stable). The fact that correct predictions of instability are generally found implies that so long as the decay is energetically possible, then it does occur. Thus, in the case of these weak-force mediated decays, the necessary condition for instability based on energy requirements appears to also be a sufficient condition (at least to a fairly good approximation). This contrasts with what we found for fission in Sections 2.3.1, 2.3.2, 2.3.3.

The only cases where the liquid drop model for the binding energy, Equ.(2.1), fails to give the correct indication of stability/instability are shown in blue. These errors are due to the limited accuracy of Equ.(2.1) for these isotopes (or their daughters). Actual binding energy data is consistent with the required inequalities. Hence, of the 40 randomly chosen isotopes examined, Equ.(2.1) gave the correct indication of stability/instability against beta decay or inverse beta decay in 37 cases, i.e. 92%. Whilst not perfect, this is a sufficiently good indication for our anthropic purposes (see 2.3.5). Note, in particular, that the method seems to perform well regarding predictions of double beta or double inverse beta decays modes, albeit as judged from a rather limited number of examples.

2.3.5 Instability Against Electron Capture

Nuclei may capture an atomic electron and so convert a proton into a neutron, providing the increase in nuclear binding energy more than compensates for the mass deficit $M_n - M_p - m_e = 0.782$ MeV. Electron capture is energetically favoured over positron emission because the latter must 'pay' for the emitted positron, rather than being given an electron as a gift, i.e. the binding energy increase for positron emission

must exceed $M_n - M_p + m_e = 1.804$ MeV, as per (2.13). Explicitly, electron capture may be written,

$$ {}^{A}_{Z}X + Ze \rightarrow {}^{A}_{Z-1}Y + (Z-1)e + \nu_e \qquad (2.20) $$

The conservation of mass-energy, and the fact that the kinetic energy of the particles on the right cannot be negative, requires,

$$ ZM_p + NM_n - B(Z,A) + Zm_e - B_e(Z) > (Z-1)M_p + (N+1)M_n + (Z-1)m_e - B(Z-1,A) - B_e(Z-1) \qquad (2.21) $$

where $B_e(Z)$ is the total electron binding energy for a neutral atom with Z electrons. Note, however, that although $B_e(Z\text{-}1)$ also represents the total electron binding energy for a neutral atom with N-1 electrons, these electrons are in an unstable configuration. This is because it will usually be an inner electron which is captured – most often from the K-shell (n = 1, L = 0). Thus, $B_e(Z\text{-}1)$ is actually a far smaller binding energy than it would be for the stable atom with Z-1 electrons. Whilst the outer electrons have binding energies of a few eV, or tens of eV, and hence negligible compared with nuclear energies, the innermost electrons can have energies of order 100 keV and cannot be ignored. For this reason the electron binding energy difference, $\Delta B_e = B_e(Z) - B_e(Z\text{-}1)$ is significant.

Inserting the values $M_p - M_n = 1.292$ MeV and $m_e = 0.511$ MeV, electron capture is energetically possible if,

$$ B(Z-1,A) - B(Z,A) > M_n - M_p - m_e + \Delta B_e = 0.782\,\text{MeV} + \Delta B_e \qquad (2.22) $$

This decay mode will be possible whenever positron emission is possible, from (2.13). However, if the nuclear binding energy difference is below 1.804MeV, positron emission is not possible and only electron capture can be active. Evans (Ch.17, Fig.3.9) shows that, where both decay modes are energetically possible, electron capture is strongly favoured in the heavier elements. Thus, assuming a positron kinetic energy equal to $m_e c^2$ for illustration, the branching ratio favours electron capture for $Z > 18$ and favours positron emission only for $Z < 18$. For $Z \sim 80$, electron capture is about 1000 times more frequent than positron emission. Thus, heavier elements tend not to be strong positron emitters.

2.3.6 A Prescription for Nuclear Stability in Alternative Universes

For the purposes of investigating the anthropic hypothesis, we need to be able to calculate whether a nucleus is stable when the strong nuclear force and/or the electromagnetic force are changed in strength. We conclude from the preceding sub-sections that there are two criteria,

1) A change in the *relative* strength of the nuclear and electromagnetic forces may prejudice nuclear stability against fission, i.e. against breaking into parts which each carry a reduced total number of nucleons (thus reducing A for the daughter nuclei).

2) A change in the absolute strength of the nuclear and/or electromagnetic forces may prejudice stability against decays mediated by the weak nuclear force, i.e.

beta decay, positron emission or electron capture (for which A remains unchanged).

In (2), the strong nuclear and electromagnetic forces may both change by the same proportion. This will be assumed for simplicity in the treatment here, any relative change in their strength being addressed under (1). In referring to the 'absolute' strengths of the nuclear and electromagnetic forces, in (2), we really mean their strength (in energy terms) relative to the nucleon mass deficit, $\Delta_N = M_n - M_p - m_e$.

2.3.6.1 Stability Against Fission

We have already proposed a simple criterion for stability against fission, in the most general sense of any A-changing particle emission. This can be represented adequately by the simple average of the symmetrical fission and deformation instability criteria. Thus, if we change the strength of the strong nuclear force contribution to the binding energy by a factor λ_s and that of the electromagnetic force by a factor λ_{em} then the criterion for stability is,

$$Z < Z_0 \quad \text{where, } Z_0 = (Z_1 + Z_2)/2 \qquad \text{and} \qquad \frac{Z_1^2}{A} = 2\frac{\lambda_s a_s}{\lambda_c a_c} = 50\frac{\lambda_s}{\lambda_c} \qquad (2.3.6.1.1)$$

and Z_2 is the smallest atomic number such that $2B(Z_2/2, A/2) > B(Z_2, A)$, where A is chosen to maximise stability for a given Z, and the binding energy is approximated by,

$$B(Z,A) = \lambda_s\left\{a_v A - a_s A^{2/3} - a_a\frac{(A-2Z)^2}{A} - S_{NZ}\frac{a_p}{A^{3/4}}\right\} - \lambda_{em} a_c \frac{Z^2}{A^{1/3}} \qquad (2.3.6.1.2)$$

We see that both Z_1 and Z_2 remain unchanged if the strong nuclear and electromagnetic forces change in proportion, i.e. if $\lambda_s = \lambda_{em}$. Thus, instability against fission depends only upon the *relative* strength of the nuclear and electric forces. For the sake of exposition, we shall assume in this Section that the strong force is unchanged and that the electromagnetic contribution to the binding energy is increased by a factor $\lambda = \lambda_{em}/\lambda_s$. Bear in mind that, as far as fissile stability is concerned, this may equally be interpreted as an unchanged electromagnetic force and a strong nuclear force reduced in strength by x $1/\lambda$.

A complication in applying criteria (2.3.6.1.1-2) which is usually glossed over in books and papers is that, in a universe with $\lambda \neq 1$, the value of A/Z for the most stable nuclides will differ from the ratio in this universe. Hence, before attempting to find Z_0 we need to find A(Z) which maximises the binding energy for a given Z.

There is actually another, simpler, criterion for nuclear stability which we have not mentioned yet. This is that the binding energy must be positive, i.e. $B(Z,A) > 0$. Strangely, the likes of Barrow & Tipler, and P.C.W.Davies (1972), do not refer to this, though it appears to be a more restrictive condition than distortional instability, as we shall see below.

We shall proceed by considering some of the key biological elements in turn: carbon, nitrogen, oxygen, phosphorus, calcium and iron. For each of these we will determine

the atom mass, A, which maximises the binding energy, B(λ,Z,A), for some relevant value(s) of λ. We shall then increase λ until even this largest binding energy reduces to zero. This determines a value λ_0 such that for $\lambda \geq \lambda_0$ the element is definitely unstable, having negative binding energy for any A. We then examine the fission stability criteria for the identified values of λ, Z and A to see whether fission may provide a more restrictive stability limit than B > 0.

Carbon

Carbon is, of course, defined by Z = 6. For λ = 5 the maximum binding energy obtains for A = 14. For λ = 8 or 9 the maximum binding energy obtains for A = 16. For λ approaching 12 the maximum binding energy obtains for A = 18. For A = 18 this maximum binding energy reduces to zero for λ = 12.066. This is the upper limit of stability for carbon.

Considering next the sufficient condition for fissile instability, Z^2/A is $6^2/18 = 2.0$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 4.14$. This does not indicate instability with respect to distortion. Look now at the necessary condition for instability, 2B(Z/2, A/2) > B(Z, A). With λ = 12.066 we have B(3,9) = 0.441 MeV per nucleon. Since this is positive, it means that this criterion for fissile instability is met. We have seen that fissile instability actually results somewhere midway between the necessary and sufficient criteria. Since we are indeed between the two criteria, meeting one but not the other, we conclude that the identified value of λ = 12.066 is a reasonable estimate of the stability limit, i.e. fissile instability does not obviously provide a more restrictive limit. It *might* be that carbon would be unstable to fission in a world with λ smaller than 12.066, since fission would be permitted as regards energy conservation. But we cannot claim instability with confidence for λ smaller than 12.066 since the sufficient condition for fissile instability is *not* met. However, a carbon nucleus would definitely be unstable in a world with λ = 12.066 since it has no binding energy at all! This is true in as far as our estimate based on the liquid drop model for binding energies, (2.3.6.1.2), is indicative.

How small must λ be before carbon is definitely stable against fission, i.e. symmetrical fission would violate the energy requirements? The answer to this appears to be not very much less than λ = 12.066. For example, Z = 6 and A = 16 is stable against symmetrical fission for λ = 11.75. Consequently we will not explore this question for other elements.

Nitrogen

Z =7: The binding energy is maximum for λ approaching 11 for A = 21. This maximum is zero when λ = 11.0995. Considering the sufficient condition for fissile instability, Z^2/A is $7^2/21 = 2.33$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 4.5$. This does not indicate instability. Turning to the necessary condition for fissile instability we find that with λ = 11.0995**,** B(3,9) = 0.771 MeV/nucleon and B(4,12) = 1.185 MeV/ nucleon. Since these are positive, fission would be possible whilst conserving energy and hence cannot be definitely ruled out, but cannot be definitely predicted either. This pattern will be repeated below for the other elements. As for carbon, we conclude that the upper limit on λ for stability of the nitrogen nucleus is obtained from the criterion B > 0 and is λ < 11.0995.

Oxygen

Z = 8: The binding energy is maximum for λ around 10 for A = 24. This maximum is zero when $\lambda = 10.778$. Considering the sufficient condition for fissile instability, Z^2/A is $8^2/24 = 2.67$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 4.64$. This does not indicate instability. Turning to the necessary condition for fissile instability we find that with $\lambda = 10.778$**,** B(4,12) = 1.318 MeV/nucleon. Since this is positive, fission would be possible whilst conserving energy and hence cannot be definitely ruled out, but cannot be definitely predicted either. As for carbon and nitrogen, we conclude that the upper limit on λ for stability of the oxygen nucleus is obtained from the criterion B > 0 and is $\lambda < 10.778$.

Phosphorus

Z = 15: The binding energy is maximum for λ around 8 for A = 49. This maximum is zero when $\lambda = 8.2187$. Considering the sufficient condition for fissile instability, Z^2/A is $15^2/49 = 4.59$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 6.08$. This does not indicate instability. Turning to the necessary condition for fissile instability we find that with $\lambda = 8.2187$**,** B(7,23) = 1.491 MeV/nucleon and B(8,26) = 1.499 MeV/nucleon. Since these are positive, fission would be possible whilst conserving energy and hence cannot be definitely ruled out, but cannot be definitely predicted either. As for carbon, nitrogen and oxygen, we conclude that the upper limit on λ for stability of the phosphorus nucleus is obtained from the criterion B > 0 and is $\lambda < 8.2187$.

Calcium

Z = 20: The binding energy is maximum for λ around 7 for A = 66 or 68. This maximum is zero when $\lambda = 7.2237$. Considering the sufficient condition for fissile instability, Z^2/A is $20^2/68 = 5.88$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 6.92$. This does not indicate instability, although the margin is now becoming small. Turning to the necessary condition for fissile instability we find that with $\lambda = 7.2237$, B(10,34) = 1.681 MeV/nucleon. Since this is positive, fission would be possible whilst conserving energy and hence cannot be definitely ruled out, but cannot be definitely predicted either. As for carbon, nitrogen, oxygen and phosphorus, we conclude that the upper limit on λ for stability of the calcium nucleus is obtained from the criterion B > 0 and is $\lambda < 7.2237$. However, the smallness of the margin against the sufficient condition for fissile instability suggests that the calcium nucleus would actually be unstable for smaller values of λ.

Iron

Z = 26: The binding energy is maximum for λ around 6 for A = 88 or 90. This maximum is zero when $\lambda = 6.3577$. Considering the sufficient condition for fissile instability, Z^2/A is $26^2/90 = 7.5$ whereas the RHS of (2.3.6.1.1) is $50/\lambda = 7.86$. This does not indicate instability, although the margin is very small. Turning to the necessary condition for fissile instability we find that with $\lambda = 6.3577$, B(13,45) = 1.755 MeV/nucleon. Since this is positive, fission would be possible whilst conserving energy and hence cannot be definitely ruled out, but cannot be definitely predicted either. As for carbon, nitrogen, oxygen, phosphorus and calcium, we conclude that the upper limit on λ for stability of the calcium nucleus is obtained from the criterion B > 0 and is $\lambda < 6.3577$. However, the smallness of the margin against

the sufficient condition for fissile instability suggests that the iron nucleus would actually be unstable for smaller values of λ.

Summary Table for Stability Against Fission / Break-Up

Element (Z)	Stability Limit (λ)
Carbon (6)	12.066
Nitrogen (7)	11.100
Oxygen (8)	10.778
Phosphorus (15)	8.219
Calcium (20)	7.224
Iron (26)	6.358

The trend of decreasing tolerance to increased values of λ as Z increases is apparent. This trend continues to higher Z. At polonium (Z = 84) the limit is λ = 1, since all elements from polonium onwards are unstable in this universe.

2.3.6.2 Stability Against Weak Decays

Potential instability against weak decays requires one of the criteria (2.10), (2.13), (2.16), (2.19) or (2.22) to be obeyed. Stability against weak decays requires the reversed inequalities to *all* be obeyed, although (2.13) can be ignored in favour of (2.22) as the stronger condition. For simplicity we shall ignore the double decay modes. Reversing (2.10) and (2.22), we are then left with the criteria for *stability* as,

both, $$B(Z,A) - B(Z+1,A) > M_n - M_p - m_e = 0.782\,\text{MeV} \qquad (2.23)$$

and, $$B(Z-1,A) - B(Z,A) < M_n - M_p - m_e + \Delta B_e = 0.782\,\text{MeV} + \Delta B_e \qquad (2.24)$$

A sufficient condition for stability against electron capture results from ignoring the electron binding energy difference, ΔB_e in (2.24).

Our objective will be to estimate whether, on changing the strength of the nuclear and electromagnetic forces, a given element (Z) has *any* stable isotopes. We shall make the assumption that it suffices to consider the isotope with the largest binding energy, i.e. the A such that B(Z,A) is greatest.

If the contribution of the strong nuclear force to the binding energy changes by a factor λ_s and the electromagnetic contribution changes by a factor λ_{em} then the liquid drop model for the nuclear binding energy becomes,

$$B(Z,A) = \lambda_s \left\{ a_v A - a_s A^{2/3} - a_a \frac{(A-2Z)^2}{A} - S_{NZ} \frac{a_p}{A^{3/4}} \right\} - \lambda_{em} a_c \frac{Z^2}{A^{1/3}} \qquad (2.25)$$

For the investigation of this sub-section, i.e. stability to weak decays, we shall assume that the strong nuclear and electromagnetic contributions change by the same factor, i.e., $\lambda_s = \lambda_{em} = \lambda$. Thus, the isotope with the greatest binding energy for any given element is unchanged from that in this universe, since the factor λ changes only the absolute energy scale not its functional dependence on Z and A. Our initial discussion therefore concentrates on the most abundant/stable isotopes in this universe and

examines what the factor λ must be to render them unstable against either beta decay or electron capture.

To reduce the number of elements which we must consider, we shall concentrate on those elements most important for terrestrial life, including only a sample of the trace elements. The most abundant isotopes are (ignoring hydrogen),

$^{12}_{6}C$ $^{14}_{7}N$ $^{16}_{8}O$ $^{19}_{9}F$ $^{23}_{11}Na$ $^{24}_{12}Mg$ $^{31}_{15}P$ $^{32}_{16}S$ $^{35}_{17}Cl$ $^{39}_{19}K$ $^{40}_{20}Ca$
($^{44}_{20}Ca$) $^{56}_{26}Fe$ $^{63}_{29}Cu$ $^{64}_{30}Zn$ $^{98}_{42}Mo$ $^{127}_{53}I$

Two different calcium isotopes are included since $^{40}_{20}Ca$ is actually unstable to double ε decay, despite being by far the commonest isotope, i.e. its half-life is extremely long. Some elements have many stable isotopes, e.g. molybdenum has 7. The values of B(Z) – B(Z+1) derived from the liquid drop model, (2.1), are listed below for these isotopes, together with the factor λ which would result in their instability to beta decay:-

	[Equ(2.1)] B(Z)-B(Z+1)	λ (nucleus unstable for smaller values)	
$^{12}_{6}C$	22.323	0.035	
$^{14}_{7}N$	1.933	0.404	(spurious – see below)
$^{16}_{8}O$	19.090	0.041	
$^{19}_{9}F$	5.056	0.155	
$^{23}_{11}Na$	5.743	0.136	
$^{24}_{12}Mg$	16.284	0.048	
$^{31}_{15}P$	7.006	0.112	
$^{32}_{16}S$	15.322	0.051	
$^{35}_{17}Cl$	7.599	0.103	
$^{39}_{19}K$	8.167	0.096	
$^{40}_{20}Ca$	15.096	0.052	
$^{44}_{20}Ca$	5.702	0.137	
$^{56}_{26}Fe$	8.025	0.097	
$^{63}_{29}Cu$	4.505	0.174	
$^{64}_{30}Zn$	9.344	0.084	
$^{98}_{42}Mo$	2.666	0.293	
$^{118}_{50}Sn$	2.832	0.276	
$^{127}_{53}I$	1.445	0.541	(Actual binding energies used, not Equ.2.1)

(Data in green are examined in more detail below using *actual* binding energies)

The heavier elements are generally more sensitive to changes in the strength of the strong nuclear and electromagnetic forces. This would be even more marked for the elements beyond iodine. There is little point in pursuing this because, beyond

polonium, they are unstable to fission in any case, and, moreover, the heavier elements do not appear to be essential even for terrestrial life.

Of the elements examined above, and with the exception of iodine, nitrogen *appears* to be the most sensitive to an increase in the strength of the strong nuclear and electromagnetic forces as regards becoming unstable to beta decay. We shall therefore take a closer look at nitrogen. We will see that this sensitivity is spurious and due to the limited accuracy of Equ.(2.1). However, it is useful to examine nitrogen closely to see if other isotopes might be stable when nitrogen-14 becomes unstable. For this purpose we shall use actual binding energy data rather than Equ.(2.1). We shall also look at both beta decay and electron capture, assuming the sufficient conditions for stability: B(Z) - B(Z+1) > 0.782MeV and B(Z-1) – B(Z) < 0.782MeV. The results are summarised as follows:-

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
$^{11}_{7}$N	39.261*	>0.020	15.092	<0.052
$^{12}_{7}$N	15.492	>0.050	18.120	<0.043
$^{13}_{7}$N	18.548	>0.042	3.003	<0.260
$^{14}_{7}$N	5.825	>0.134	0.626	<1.25
$^{15}_{7}$N	3.536	>0.221	<0	stable
$^{16}_{7}$N	<0	unstable	<0	stable
$^{17}_{7}$N	<0	unstable	<0	stable

** Equ.(2.1) used (actual data not found)*

Note that the beta decay mass difference for nitrogen-14 in the above Table is much greater than that derived from Equ.(2.1) in the preceding list. Hence, we conclude that the particular sensitivity of nitrogen to changes in the force strength was spurious and resulted from the inaccuracy of Equ.(2.1) in this case. Nevertheless nitrogen is still amongst the more sensitive of the light elements to changes in the strength of the nuclear and electrostatic force.

The above Table correctly indicates that nitrogen-11, 12, 13, 16 and 17 are unstable (in this universe), the first three to positron emission or EC, and the last two to beta decay. There are two stable isotopes of nitrogen, 14 and 15, as indicated by the Table. Note that the most abundant isotope, nitrogen-14, rather narrowly misses being unstable to electron capture. (In fact, Equ.2.1 would give a false indication of instability, and so actual binding energy data had to be used).

Nitrogen-14 is stable so long as $0.134 < \lambda < 1.25$. Nitrogen-15 is stable so long as $\lambda > 0.221$. Nitrogen-16 and 17 are always unstable, as is nitrogen-12, irrespective of how the strength of the forces are scaled (as long as they vary in proportion). However, there appear to be strength values for which nitrogen-11 and/or nitrogen-13 become stable (despite being unstable in this universe). Thus, nitrogen-11 is stable for $0.020 < \lambda < 0.052$, and nitrogen-13 is stable for $0.042 < \lambda < 0.260$. Strictly we have not checked that they are also stable against double decay modes. For completeness we check this next:-

Stability of $^{11}_{7}\mathrm{N}$ and $^{13}_{7}\mathrm{N}$ to Double Beta Decay:-

Since these nuclei have an excess of protons they will not be susceptible to beta decays. Looking at the binding energy differences confirms this.

Stability of $^{11}_{7}\mathrm{N}$ and $^{13}_{7}\mathrm{N}$ to Double Positron Emission or Double Electron Capture:-

I'm not sure if double electron capture is possible, but if so, stability is assured against it, and also against double positron emission, if $B(Z-2) - B(Z) < 1.564\text{MeV}$. Double electron capture is impossible, but double positron emission possible, if this binding energy difference is between 1.564MeV and 3.608MeV. For nitrogen-11 this binding energy difference evaluates, using Equ.(2.1), to 24.899MeV. But, for stability against single electron capture we are assuming that the force strengths are scaled down by $\lambda < 0.052$ (see above Table). Hence, the binding energy difference for double decays becomes < 1.295MeV. Hence, both double decays modes are stable. For nitrogen-13 the double decay binding energy difference is negative, and hence nitrogen-13 is stable against double decays even without scaling the forces.

If $\lambda > 1.25$ all nitrogen isotopes are unstable except for nitrogen-15. It is easily checked that nitrogen-15 is stable against all double decay modes (and this remains true even if λ is reduced to 0.221 or takes arbitrarily large values).

In conclusion (for Nitrogen): The most abundant isotope of nitrogen, $^{14}_{7}\mathrm{N}$, is stable so long as $0.134 < \lambda < 1.25$. However, nitrogen-15 is stable for arbitrarily large λ so long as $\lambda > 0.221$. For smaller values of λ, nitrogen-11 is stable for $0.020 < \lambda < 0.052$, and nitrogen-13 is stable for $0.042 < \lambda < 0.260$. Thus there is at least one stable isotope of nitrogen for any $\mathbf{\lambda > 0.020}$. Stability has been confirmed with respect to all weak decay modes, double and single, and including electron capture. We conclude that the strong nuclear force and the electromagnetic force, if varied in proportion, would have to reduce their contribution to the binding energy to below 2% of their actual value for *all* nitrogen isotopes to become unstable to weak decays.

However, it is worth remarking that the potential existence of a stable isotope may not be enough. There also has to be a means by which it can be produced, presumably in stars. For example, the abundance of nitrogen-15 is 0.37%. Hence, unless some new reaction pathway to form $^{15}_{7}\mathrm{N}$ comes about as a consequence of the changes in the force strengths, then nitrogen might be reduced in abundance by a factor of about 0.0037. If the lifetime of $^{14}_{7}\mathrm{N}$ were sufficiently short it would play no part in stellar fusion and nucleosynthesis. For example, the CN sequence would be subverted, though it may be replaced by something equivalent. As regards nuclear heating, this does not seem too profound a problem, since, at worst, the pp sequences are still available. The implications of having no $^{14}_{7}\mathrm{N}$ on stellar nucleosynthesis would require more careful scrutiny.

For completeness we should carry out a similar detailed scrutiny of the stability of a wide range of isotopes for each biologically important element. This becomes more laborious for heavier elements because a larger number of isotopes must be considered. Also, it is not clear which are the truly crucial elements. Can we possibly do without sodium or chlorine? Who knows? The four most obvious biological

elements are hydrogen, carbon, nitrogen and oxygen. The stability of the most abundant isotope of carbon and oxygen appears to require $\lambda > 4\%$ or so. Exploring whether other isotopes are still stable for smaller λ hardly seems worthwhile since, together with the detailed examination of nitrogen, we have already established that a reduction in the force strengths of nearly two orders of magnitude might be accommodated before the most obviously key elements become unstable to weak decays. It is true that there may be subtleties which render some elements more vulnerable to smaller changes. But if this affects only one or two elements (other than H, C, O or N) how are we to judge whether this is terminal for life?

We have checked a few elements for stability to electron capture. For carbon, oxygen, sodium and sulphur, the binding energy difference B(Z-1) – B(Z) is negative, which means that the stability condition against electron capture is guaranteed whatever the value of λ. We have chosen to look more closely at chlorine, since this appears to be borderline with respect to electron capture. For this purpose we again made use of actual binding energy data, rather than Equ.(2.1), since the latter gave a false indication of chlorine-35 being unstable to electron capture.

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
$^{32}_{17}$Cl	11.934	>0.065	13.468	<0.058
$^{33}_{17}$Cl	12.404	>0.063	6.365	<0.122
$^{34}_{17}$Cl	6.845	>0.114	6.274	<0.125
$^{35}_{17}$Cl	6.747	>0.116	0.615	<1.27
$^{36}_{17}$Cl	0.0738	>10.6	1.924	<0.406
$^{37}_{17}$Cl	1.596	>0.490	<0	stable
$^{38}_{17}$Cl	<0	unstable	<0	stable
$^{39}_{17}$Cl	<0	unstable	<0	stable

Thus, $^{32}_{17}$Cl, $^{36}_{17}$Cl, $^{38}_{17}$Cl and $^{39}_{17}$Cl are unstable for any value of λ.

$^{33}_{17}$Cl is stable for $0.063 < \lambda < 0.122$;

$^{34}_{17}$Cl is stable for $0.114 < \lambda < 0.125$;

$^{35}_{17}$Cl is stable for $0.116 < \lambda < 1.27$;

$^{37}_{17}$Cl is stable for $0.490 < \lambda$.

Thus, the situation is very similar to that for nitrogen. As long as **$\lambda > 0.063$** there is at least one stable isotope of chlorine. (Although we have not examined stability to double decay modes).

I really need to carry out this more detailed study for ~~C~~, O, ~~Na~~, F, P and Ca. I'll do it some day!!

Next we repeat the above analysis for carbon (again based on actual binding energies), yielding:-

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
${}^{9}_{6}C$	~39*	>0.02*	17.280	<0.045
${}^{10}_{6}C$	24.782	>0.032	4.430	<0.177
${}^{11}_{6}C$	15.092	>0.052	2.765	<0.283
${}^{12}_{6}C$	18.121	>0.043	<0	stable
${}^{13}_{6}C$	3.003	>0.260	<0	stable
${}^{14}_{6}C$	0.626	>1.249	<0	stable
${}^{15}_{6}C$	<0	unstable	<0	stable
${}^{16}_{6}C$	<0	unstable	<0	stable

** B(7,9) was not available and was assumed to be ~0*

From which we conclude that 15C and 16C are always unstable, but that at least one of the other carbon isotopes is stable for any $\lambda > 0.02$. Specifically, the isotopes are stable for…

${}^{9}_{6}C$	$0.02 < \lambda < 0.045$
${}^{10}_{6}C$	$0.032 < \lambda < 0.177$
${}^{11}_{6}C$	$0.052 < \lambda < 0.283$
${}^{12}_{6}C$	$0.043 < \lambda$
${}^{13}_{6}C$	$0.260 < \lambda$
${}^{14}_{6}C$	$1.249 < \lambda$

(Strictly we have not checked stability against double decays and captures).

Next we repeat the above analysis for sodium (again based on actual binding energies), yielding:-

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
${}^{20}_{11}Na$	11.508	>0.068	14.669	<0.053
${}^{21}_{11}Na$	13.878	>0.056	4.330	<0.181
${}^{22}_{11}Na$	5.568	>0.141	3.625	<0.216
${}^{23}_{11}Na$	4.839	>0.162 ${}^{21}_{11}Na$	<0	stable
${}^{24}_{11}Na$	<0	unstable	<0	stable
${}^{25}_{11}Na$	<0	unstable	<0	stable
${}^{26}_{11}Na$	<0	unstable	<0	stable

Hence we see that ${}^{20}_{11}Na$, ${}^{24}_{11}Na$, ${}^{25}_{11}Na$ and ${}^{26}_{11}Na$ are always unstable, but at least one stable isotope exists so long as **$\lambda > 0.056$**. Specifically the isotopes are stable for:-

${}^{21}_{11}Na$	$0.056 < \lambda < 0.181$
${}^{22}_{11}Na$	$0.141 < \lambda < 0.216$
${}^{23}_{11}Na$	$0.162 < \lambda$

Next we repeat the above analysis for fluorine (again based on actual binding energies), yielding:-

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
$^{16}_{9}F$	14.094	>0.055	16.200	<0.048
$^{17}_{9}F$	15.316	>0.051	3.542	<0.221
$^{18}_{9}F$	5.216	>0.150	2.438	<0.321
$^{19}_{9}F$	4.021	>0.195	<0	stable
$^{20}_{9}F$	<0	unstable	<0	stable
$^{21}_{9}F$	<0	unstable	<0	stable
$^{22}_{9}F$	<0	unstable	<0	stable

Thus $^{16}_{9}F$, $^{20}_{9}F$, $^{21}_{9}F$ and $^{22}_{9}F$ are always unstable, but at least one stable isotope of fluorine exists so long as $\lambda > 0.051$. Specifically the stable regimes are:-

$^{17}_{9}F$ $0.051 < \lambda < 0.221$

$^{18}_{9}F$ $0.150 < \lambda < 0.321$

$^{19}_{9}F$ $0.195 < \lambda$

Next we repeat the above analysis for phosphorus (again based on actual binding energies), yielding:-

	B(Z)-B(Z+1)	λ_{stable}	B(Z-1)-B(Z)	λ_{stable}
$^{27}_{15}P$	14.094	>0.055	16.200	<0.048
$^{28}_{15}P$	15.316	>0.051	3.542	<0.221
$^{29}_{15}P$	5.216	>0.150	2.438	<0.321
$^{30}_{15}P$	4.021	>0.195	<0	stable
$^{31}_{15}P$	<0	unstable	<0	stable
$^{32}_{15}P$	<0	unstable	<0	stable
$^{33}_{15}P$	<0	unstable	<0	stable

Thus $^{28}_{15}P$ and $^{32}_{15}P$ are always unstable, but at least one stable isotope of phosphorus exists so long as $\lambda > 0.041$. Specifically the stable regimes are:-

$^{27}_{15}P$ $0.041< \lambda < 0.063$

$^{29}_{15}P$ $0.054 < \lambda < 0.137$

$^{30}_{15}P$ $0.113 < \lambda < 0.156$

$^{31}_{15}P$ $0.127 < \lambda$

$^{33}_{15}P$ $1.464 < \lambda$

In summary, at least one stable isotope of the following elements occurs provided that λ exceeds the stated value:-

Summary for Strong Nuclear Force and Electrostatic Force Varied in Proportion

Element	**Z**	**At Least One Stable Isotope if λ Exceeds:-**
C	6	0.02
N	7	0.02
O	8	~0.041*
F	9	0.051
Na	11	0.056
Mg	12	~0.048*
P	15	0.041
S	16	~0.051*
Cl	17	0.063
Ca	20	~0.052*

*Based on Equ.(2.1) analysis of the most stable isotope in this universe only.

2.3.6.2.1 Varying the Nuclear and Electric Forces Inversely

In the last sub-section we chose to vary the strength of the strong nuclear and electrostatic forces in proportion. In this sub-section we vary the two forces in inverse proportion, i.e. we assume $\lambda_s = 1/\lambda_{em} = \lambda$. This will lead to a greater change in the binding energy for a given λ. For example, if λ > 1 the nuclear force is greater in strength but the electric force is weaker, and both these changes cause the binding energy to increase. Unlike in the last sub-section, we cannot simply scale binding energies by λ. Instead the binding energy is given by,

$$B(Z,A) = \lambda B_s(Z,A) - B_c(Z,A)/\lambda \qquad (?)$$

where B_c is the Coulomb energy, which contributes negatively to the binding energy, and B_s is the total of the remaining contributions to the binding energy, i.e. the strong nuclear force parts. The Coulomb part is given by,

$$B_c(Z,A) = a_c \frac{Z^2}{A^{1/3}}, \text{ where } a_c = 0.71 \text{ MeV}. \qquad (?)$$

The strong-force binding energy Bs is found from the known total binding energy when λ = 1. Hence, knowing Bc and Bs we can find the binding energy in our alternative universe from (?). We illustrate the process for fluorine:-

Consider ^{17}F:
(a)Electron capture:
$B_c(8,17)$ = 17.672 MeV and hence $B_s(8,17)$ = 149.434 MeV.
$B_c(9,17)$ = 22.366 MeV and hence $B_s(9,17)$ = 150.586 MeV.
Hence: $B_c(8,17) - B_c(9,17) = [149.434\lambda - 17.672/\lambda] - [150.586\lambda - 22.366/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,

$-1.152\lambda^2 - 0.782\lambda + 4.694 = 0$, and hence $\underline{\lambda > 1.708}$ ensures that the stability criterion against electron capture $B_c(8,17) - B_c(9,17) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(10,17) = 27.613$ MeV and hence $B_s(10,17) = 140.517$ MeV.
Hence: $B_c(9,17) - B_c(10,17) = [150.586\lambda - 22.366/\lambda] - [140.517\lambda - 27.613/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$10.069\lambda^2 - 0.782\lambda + 5.247 = 0$, for which there is no solution.
The stability criterion against beta decay, $B_c(9,17) - B_c(10,17) > 0.782$ MeV, reduces to $10.069\lambda + 5.247/\lambda > 0.782$ MeV, and hence is respected for $\lambda = 1$. It follows that it is respected for all values of λ, and hence fluorine-17 is stable against beta decay for all values of λ.

Consider ^{18}F:
(a)Electron capture:
$B_c(8,18) = 17.339$ MeV and hence $B_s(8,18) = 157.146$ MeV.
$B_c(9,18) = 21.944$ MeV and hence $B_s(9,18) = 159.313$ MeV.
Hence: $B_c(8,18) - B_c(9,18) = [157.146\lambda - 17.339/\lambda] - [159.313\lambda - 21.944/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-2.167\lambda^2 - 0.782\lambda + 4.605 = 0$, and hence $\underline{\lambda > 1.288}$ ensures that the stability criterion against electron capture $B_c(8,18) - B_c(9,18) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(10,18) = 27.092$ MeV and hence $B_s(10,18) = 159.245$ MeV.
Hence: $B_c(9,18) - B_c(10,18) = [159.313\lambda - 21.944/\lambda] - [159.245\lambda - 27.092/\lambda]$

Equating this to the limiting value of 0.782 MeV for stability gives,
$0.068\lambda^2 - 0.782\lambda + 5.148 = 0$, for which there is no solution.
The stability criterion against beta decay, $B_c(9,18) - B_c(10,18) > 0.782$ MeV, reduces to $0.068\lambda + 5.148/\lambda > 0.782$ MeV, and hence is respected for $\lambda = 1$. It follows that it is respected for all values of λ, and hence fluorine-19 is stable against beta decay for all values of λ.

Consider ^{19}F:
(a)Electron capture:
$B_c(8,19) = 17.029$ MeV and hence $B_s(8,19) = 160.791$ MeV.
$B_c(9,19) = 21.552$ MeV and hence $B_s(9,19) = 169.353$ MeV.
Hence: $B_c(8,19) - B_c(9,19) = [160.791\lambda - 17.029/\lambda] - [169.353\lambda - 21.552/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-8.562\lambda^2 - 0.782\lambda + 4.523 = 0$, and hence $\lambda > 0.683$ ensures that the stability criterion against electron capture $B_c(8,19) - B_c(9,19) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(10,19) = 26.608$ MeV and hence $B_s(10,19) = 170.388$ MeV.
Hence: $B_c(9,19) - B_c(10,19) = [169.353\lambda - 21.552/\lambda] - [170.388\lambda - 26.608/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-1.035\lambda^2 - 0.782\lambda + 5.056 = 0$, which gives $\lambda < 1.864$ in oder to respect the stability criterion against beta decay, i.e., $B_c(9,19) - B_c(10,19) > 0.782$ MeV.
Hence, overall, fluorine-19 is stable if $\underline{0.683 < \lambda < 1.864}$.

Consider ^{20}F:
(a)Electron capture:

$B_c(8,20)$ = 16.740 MeV and hence $B_s(8,20)$ = 168.110 MeV.
$B_c(9,20)$ = 21.187 MeV and hence $B_s(9,20)$ = 175.590 MeV.
Hence: $B_c(8,20) - B_c(9,20) = [168.110\lambda - 16.740/\lambda] - [175.590\lambda - 21.187/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-7.480\lambda^2 - 0.782\lambda + 4.447 = 0$, and hence $\lambda > 0.721$ ensures that the stability criterion against electron capture $B_c(8,20) - B_c(9,20) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(10,20)$ = 26.157 MeV and hence $B_s(10,20)$ = 186.802 MeV.
Hence: $B_c(9,20) - B_c(10,20) = [175.590\lambda - 21.187/\lambda] - [186.802\lambda - 26.157/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-11.212\lambda^2 - 0.782\lambda + 4.970 = 0$, which gives $\lambda < 0.632$ in oder to respect the stability criterion against beta decay, i.e., $B_c(9,20) - B_c(10,20) > 0.782$ MeV.
Hence, overall, fluorine-20 is unstable for any λ because we cannot have both $\lambda > 0.721$ and $\lambda < 0.632$.

In summary for fluorine, if we vary the strong nuclear force and the electric force in inverse proportion, then fluorine isotopes are stable for the following ranges of λ:-

^{17}F	$1.708 < \lambda < \infty$
^{18}F	$1.288 < \lambda < \infty$
^{19}F	$0.683 < \lambda < 1.864$
^{20}F	Unstable

Hence there is at least one stable isotope of fluorine for any **$\lambda > 0.683$**.

We next carry out the same calculations for carbon. We find that ^{9}C and ^{10}C are always unstable against electron capture. However, stable regimes exist for ^{11}C, ^{12}C, ^{13}C, ^{14}C, ^{15}C and ^{16}C, as detailed below:-

Consider ^{11}C:
(a)Electron capture:
$B_c(5,11)$ = 7.981 MeV and hence $B_s(5,11)$ = 84.186 MeV.
$B_c(6,11)$ = 11.493 MeV and hence $B_s(6,11)$ = 84.933 MeV.
Hence: $B_c(5,11) - B_c(6,11) = [84.186\lambda - 7.981/\lambda] - [84.933\lambda - 11.493/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-0.747\lambda^2 - 0.782\lambda + 3.512 = 0$, and hence $\lambda > 1.707$ ensures that the stability criterion against electron capture $B_c(5,11) - B_c(6,11) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,11)$ = 15.643 MeV and hence $B_s(7,11)$ = 73.991 MeV.
Hence: $B_c(6,11) - B_c(7,11) = [84.933\lambda - 11.493/\lambda] - [73.991\lambda - 15.643/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$10.942\lambda^2 - 0.782\lambda + 4.150 = 0$, which has no solution. Hence the stability criterion against beta decay, i.e., $B_c(6,11) - B_c(7,11) > 0.782$ MeV, is always respected.

Consider ^{12}C:
(a)Electron capture:
$B_c(5,12)$ = 7.753 MeV and hence $B_s(5,12)$ = 87.328 MeV.
$B_c(6,12)$ = 11.164 MeV and hence $B_s(6,12)$ = 103.326 MeV.
Hence: $B_c(5,12) - B_c(6,12) = [87.328\lambda - 7.753/\lambda] - [103.326\lambda - 11.164/\lambda]$

Equating this to the limiting value of 0.782 MeV for stability gives,
$-15.998\lambda^2 - 0.782\lambda + 3.411 = 0$, and hence $\underline{\lambda > 0.438}$ ensures that the stability criterion against electron capture $B_c(5,12) - B_c(6,12) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,12) = 15.196$ MeV and hence $B_s(7,12) = 89.237$ MeV.
Hence: $B_c(6,12) - B_c(7,12) = [103.326\lambda - 11.164/\lambda] - [89.237\lambda - 15.196/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$14.089\lambda^2 - 0.782\lambda + 4.032 = 0$, which has no solution. Hence the stability criterion against beta decay, i.e., $B_c(6,12) - B_c(7,12) > 0.782$ MeV, is always respected.

Consider ^{13}C:
(a)Electron capture:
$B_c(5,13) = 7.549$ MeV and hence $B_s(5,13) = 92.002$ MeV.
$B_c(6,13) = 10.870$ MeV and hence $B_s(6,13) = 107.979$ MeV.
Hence: $B_c(5,13) - B_c(6,13) = [92.002\lambda - 7.549/\lambda] - [107.979\lambda - 10.870/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-15.976\lambda^2 - 0.782\lambda + 3.321 = 0$, and hence $\lambda > 0.432$ ensures that the stability criterion against electron capture $B_c(5,13) - B_c(6,13) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,13) = 14.796$ MeV and hence $B_s(7,13) = 108.901$ MeV.
Hence: $B_c(6,13) - B_c(7,13) = [107.979\lambda - 10.870/\lambda] - [108.901\lambda - 14.796/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-0.922\lambda^2 - 0.782\lambda + 3.926 = 0$, and hence $\underline{\lambda < 1.682}$ ensures that the stability criterion against beta decay, i.e., $B_c(6,13) - B_c(7,13) > 0.782$ MeV, is respected.
Thus, overall, ^{13}C is stable for the range $\underline{0.432 < \lambda < 1.682}$.

Consider ^{14}C:
(a)Electron capture:
$B_c(5,14) = 7.365$ MeV and hence $B_s(5,14) = 92.788$ MeV.
$B_c(6,14) = 10.605$ MeV and hence $B_s(6,14) = 115.890$ MeV.
Hence: $B_c(5,14) - B_c(6,14) = [92.788\lambda - 7.365/\lambda] - [115.890\lambda - 10.605/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-23.102\lambda^2 - 0.782\lambda + 3.240 = 0$, and hence $\lambda > 0.358$ ensures that the stability criterion against electron capture $B_c(5,14) - B_c(6,14) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,14) = 14.435$ MeV and hence $B_s(7,14) = 119.093$ MeV.
Hence: $B_c(6,14) - B_c(7,14) = [115.890\lambda - 10.605/\lambda] - [119.093\lambda - 14.435/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-3.203\lambda^2 - 0.782\lambda + 3.830 = 0$, and hence $\lambda < 0.978$ ensures that the stability criterion against beta decay, i.e., $B_c(6,14) - B_c(7,14) > 0.782$ MeV, is respected.
Thus, overall, ^{14}C is stable for the range $\underline{0.358 < \lambda < 0.978}$. Notice how closely ^{14}C misses being stable in this universe.

Consider ^{15}C:
(a)Electron capture:
$B_c(5,15) = 7.197$ MeV and hence $B_s(5,15) = 95.388$ MeV.
$B_c(6,15) = 10.364$ MeV and hence $B_s(6,15) = 116.867$ MeV.
Hence: $B_c(5,15) - B_c(6,15) = [95.388\lambda - 7.197/\lambda] - [116.867\lambda - 10.364/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,

$-21.478\lambda^2 - 0.782\lambda + 3.167 = 0$, and hence $\lambda > 0.366$ ensures that the stability criterion against electron capture $B_c(5,15) - B_c(6,15) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,15) = 14.107$ MeV and hence $B_s(7,15) = 129.599$ MeV.
Hence: $B_c(6,15) - B_c(7,15) = [116.867\lambda - 10.364/\lambda] - [129.599\lambda - 14.107/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-12.732\lambda^2 - 0.782\lambda + 3.743 = 0$, and hence $\lambda < 0.512$ ensures that the stability criterion
against beta decay, i.e., $B_c(6,15) - B_c(7,15) > 0.782$ MeV, is respected.
Thus, overall, ^{15}C is stable for the range $0.366 < \lambda < 0.512$.

Consider ^{16}C:
(a)Electron capture:
$B_c(5,16) = 7.044$ MeV and hence $B_s(5,16) = 95.192$ MeV.
$B_c(6,16) = 10.143$ MeV and hence $B_s(6,16) = 120.896$ MeV.
Hence: $B_c(5,16) - B_c(6,16) = [95.192\lambda - 7.044/\lambda] - [120.896\lambda - 10.143/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-25.705\lambda^2 - 0.782\lambda + 3.099 = 0$, and hence $\lambda > 0.332$ ensures that the stability criterion against electron capture $B_c(5,16) - B_c(6,16) < 0.782$ MeV is respected.
(b)Beta Decay:
$B_c(7,16) = 13.806$ MeV and hence $B_s(7,16) = 131.788$ MeV.
Hence: $B_c(6,16) - B_c(7,16) = [120.896\lambda - 10.143/\lambda] - [131.788\lambda - 13.806/\lambda]$
Equating this to the limiting value of 0.782 MeV for stability gives,
$-10.892\lambda^2 - 0.782\lambda + 3.663 = 0$, and hence $\lambda < 0.545$ ensures that the stability criterion
against beta decay, i.e., $B_c(6,16) - B_c(7,16) > 0.782$ MeV, is respected.
Thus, overall, ^{16}C is stable for the range $0.332 < \lambda < 0.545$.

In summary for carbon, if we vary the strong nuclear force and the electric force in inverse proportion, then fluorine isotopes are stable for the following ranges of λ:-

11C	$1.707 < \lambda < \infty$
12C	$0.438 < \lambda < \infty$
13C	$0.432 < \lambda < 1.682$
14C	$0.358 < \lambda < 0.978$
15C	$0.366 < \lambda < 0.512$
16C	$0.332 < \lambda < 0.545$

Hence there is at least one stable isotope of carbon for any $\lambda > \mathbf{0.332}$. Note that for the range $0.438 < \lambda < 0.512$ there are at least five stable isotopes of carbon.

For the remaining elements which we shall study in this way we shall give the results only, the method being exactly as above. Thus, for oxygen we find:-

^{13}O	unstable
^{14}O	unstable
^{15}O	$1.901 < \lambda < \infty$
^{16}O	$0.525 < \lambda < \infty$

^{17}O $0.555 < \lambda < 1.708$
^{18}O $0.464 < \lambda < 1.289$
^{19}O $0.479 < \lambda < 0.683$

Hence, there is at least one stable isotope of oxygen provided that **$\lambda > 0.464$**. Note that the above analysis implies that ^{16}O, ^{17}O and ^{18}O should all be stable in this universe – which is correct. Also ^{15}O decays by electron capture, as predicted, and ^{19}O decays by beta decay, as predicted.

Next, for sodium we find,

^{20}Na unstable
^{21}Na $1.908 < \lambda < \infty$
^{22}Na $1.556 < \lambda < 3.462$
^{23}Na $0.727 < \lambda < 2.126$
^{24}Na unstable
^{25}Na $0.630 < \lambda < 0.760$
^{26}Na unstable

Hence, as long as **$\lambda > 0.630$** there is at least one stable isotope of sodium. Note that the above analysis predicts that ^{23}Na should be the only stable isotope of sodium in this universe, which is correct.

Finally we consider phosphorus, for the results are:-

^{28}P unstable
^{29}P $2.249 < \lambda < \infty$
^{30}P unstable
^{31}P $0.898 < \lambda < 2.475$
^{32}P unstable
^{33}P $0.714 < \lambda < 0.981$

Hence, as long as **$\lambda > 0.714$** there is at least one stable isotope of phosphorus. Note that the above analysis implies that the only stable isotope of phosphorus in this universe is ^{31}P. which is correct.

Summary: When the strong force contribution to the binding energy is *increased* by a factor λ, and the electrostatic contribution to the binding energy is *decreased* by the same factor, elements are stable for the following ranges of λ:-

element	λ greater than
C(6)	0.332
O(8)	0.464
F(9)	0.683
Na(11)	0.630
P(15)	0.714

2.3.7 Energy Factor λ In Terms Of Strong Coupling Constant g_s

How do the factors λ_s and λ_{em} by which the strong-nuclear and electromagnetic contributions to the binding energy are increased relate to the respective coupling constants?

Consider firstly the case of the Coulomb term in the binding energy,

$$B_{em}(Z,A) = -\lambda_{em} a_c \frac{Z^2}{A^{1/3}} \qquad \text{where, } a_c = \frac{3}{5}\cdot\frac{\alpha\hbar c}{r_0} = 0.71 \qquad (2.3.7.1)$$

Thus, the Coulomb λ-factor is simply the factor by which the electromagnetic fine structure constant is increased,

EITHER: $$\lambda_{em} = \frac{\alpha}{\alpha_0} = \frac{e^2}{e_0^2} \qquad (2.3.7.2a)$$

where the subscript, $_0$, denotes the values in this universe. This assumes that the range, r_0, of the nuclear force is unchanged when the strength of the nuclear force is changed. This is reasonable if the mass of the pion is regarded as independent of the strong coupling constant, g_s, since it is the mass of the pion which determines the range of the strong force. Since the masses of the nucleons are regarded as independent of g_s in our set of universal constants, it may be reasonable to assume the same for the pion mass. On the other hand, in a Bohr-type theory (i.e. nucleons bound by a central potential) the range, or nuclear size, would be proportional to $1/g_s^2$. In this case we get,

OR: $$\lambda_{em} = \frac{g_s^2}{g_{s0}^2}\cdot\frac{\alpha}{\alpha_0} = \frac{g_s^2}{g_{s0}^2}\cdot\frac{e^2}{e_0^2} \qquad (2.3.7.2b)$$

Hence, the Coulomb binding energy factor λ_{em} is not only related to a change in the strength of the electromagnetic force but also depends on the change in the strong nuclear force. Which of (2.3.7.2a) or (2.3.7.2b) is correct is not clear, although Davies (1972) favours (2.3.7.2b). Does he mention the implied change in e, then?

The scaling factor applied to the nuclear force contribution to the binding energy is even more challenging to relate back to the coupling constant, g_s, because we have no theory of the nuclear many-body problem that allows us to derive the equivalent of (2.3.7.1). If the bound states resulted from a Bohr-type central potential, then, by analogy with atomic energy states, we would expect binding energies proportional to g_s^4. Davies (1972) argues in favour of this, but largely, it seems, in opposition to a previous paper which proposed an unsubstantiated g_s^2 dependence. Hence, our first option is to assume,

EITHER: $$\lambda_s = \frac{g_s^4}{g_{s0}^4} \qquad (2.3.7.3a)$$

However, hydrogenic type energy levels really only apply for a 1/r, Coulomb-like, infinite range potential. In contrast, the strong nuclear force has a finite range, and is thus more similar to a square potential well. The main qualitative distinction between infinite and finite range potentials appears to be that the former always have bound states no matter how small the coupling constant. Thus, a g_s^2/r potential results in binding energies which are fractions of g_s^4 times some characteristic energy scale. Thus, the binding energy is non-zero so long as g_s is non-zero. In contrast, a square potential well has no bound states at all if a^2V is less than some non-zero value. We have we have explored this in some detail already in Chapter FT1. This behaviour appears to be characteristic of finite range potentials, in that the exponential and Yukawa potentials also give rise to bound states only if their depth or range is sufficiently great. The result is that binding energies for finite range potentials are not simply proportional to the coupling strength. A better prescription, based on the square potential well, is to assume that the binding energies scale as follows,

$$\frac{B}{B_0} = \frac{\left(g_s - g_s^{min}\right)^2}{\left(g_{s0} - g_s^{min}\right)^2} \qquad (2.3.7.4)$$

where g_s^{min} is the smallest coupling constant which would just bind the nucleus in question. Thus, for a deuteron we have derived in Chapter FT1 the value $g_s^{min} = 0.85\, g_{s0}$. Since other nuclei have much larger binding energies per nucleon, we may expect g_s^{min}/g_{s0} to be rather smaller than 0.85 for other nuclei. In the case of the deuteron we were able to derive a value for g_s^{min} because the two-body problem was sufficiently simple to analyse by explicitly solving the Schrodinger equation, and effective potentials may be taken from the literature. We do not have this option for the heavier nuclei. In lieu of this we use the following crude approximation:-

Assume that the ratio of the binding energies per nucleon for deuterium and any given nucleus is given by the ration of their $\left(g_s - g_s^{min}\right)^2$ factors. Thus,

$$\frac{\left(g_s - g_s^{min}\right)^2}{\left(g_s - g_{sD}^{min}\right)^2} = \frac{B}{B_D} \qquad (2.3.7.5)$$

If we take $g_s = g_{s0}$ then we can interpret B_D as the binding energy per nucleon of the deuteron in this universe, i.e. 1.112 MeV. Take carbon, for example, then B is 7.68 MeV per nucleon. We have,

$$\frac{\left(1 - g_s^{min}/g_{s0}\right)^2}{\left(1 - g_{sD}^{min}/g_{s0}\right)^2} = \frac{\left(1 - g_s^{min}/g_{s0}\right)^2}{(1 - 0.85)^2} = \frac{7.68}{1.112} \qquad (2.3.7.6)$$

which gives $g_s^{min}/g_{s0} = 0.606$ for carbon. Repeating for other nuclei gives:-

Element (Z,A)	**B (MeV, per nucleon)**	g_s^{min} / g_{s0}
Carbon (6,12)	7.680	0.606
Nitrogen (7,14)	7.476	0.611
Oxygen (8,16)	7.976	0.598
Sodium (11,23)	8.133	0.594
Phosphorus (15,31)	8.419	0.587
Sulphur (16,32)	8.432	0.587
Chlorine (17,35)	8.497	0.585
Calcium (20,40)		
Iron (26)	8.790	0.578

We are now in a position to translate the energy factor λ_s into a factor $\tilde{\lambda}_s$ on the coupling constant, i.e. $g_s = \tilde{\lambda}_s g_{s0}$. For an equivalent affect on the binding energy we must have,

$$\left(\tilde{\lambda}_s g_{s0} - g_s^{min}\right)^2 = \lambda_s \left(g_{s0} - g_s^{min}\right)^2 \qquad (2.3.7.7)$$

which reduces to,

OR:
$$\tilde{\lambda}_s = \frac{g_s}{g_{s0}} = \lambda_s^{1/2}\left(1 - \frac{g_s^{min}}{g_{s0}}\right) + \frac{g_s^{min}}{g_{s0}} \qquad (2.3.7.3b)$$

where g_s^{min} / g_{s0} is given by the above Table. (2.3.7.3b) is an alternative to (2.3.7.3a).

3. Results

Stability Against Fission

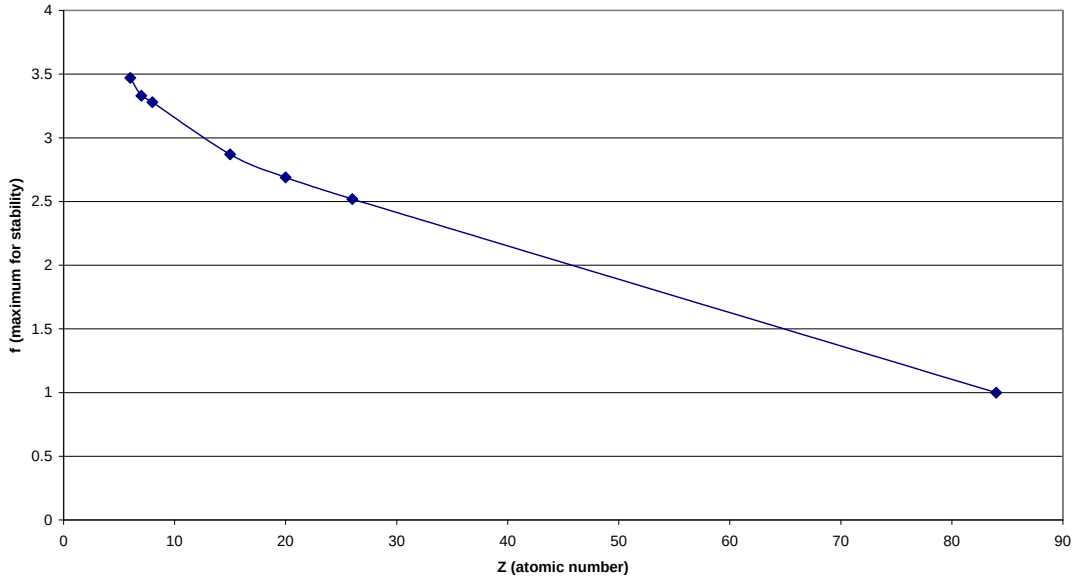
Element	$\lambda = \lambda_{em}$	$e/e_0 = \sqrt{\lambda_{em}}$	$\lambda_s = 1/\lambda$	g_s/g_{s0} [(1)]	e/e_0 [(1)]	g_s/g_{s0} [(2)]	e/e_0 [(2)]
C(6)	12.066	3.47	0.0829	0.537	1 or 1.86	0.719	1 or 1.39
N(7)	11.1	3.33	0.0901	0.548	1 or 1.82	0.728	1 or 1.37
O(8)	10.778	3.28	0.0928	0.552	1 or 1.81	0.720	1 or 1.39
P(15)	8.219	2.87	0.1217	0.591	1 or 1.69	0.731	1 or 1.37
Ca(20)	7.224	2.69	0.1384	0.610	1 or 1.64	0.742	1 or 1.35
Fe(26)	6.358	2.52	0.1573	0.630	1 or 1.59	0.745	1 or 1.34
Po(84)	1	1	1	1	1	1	1

Notes:-

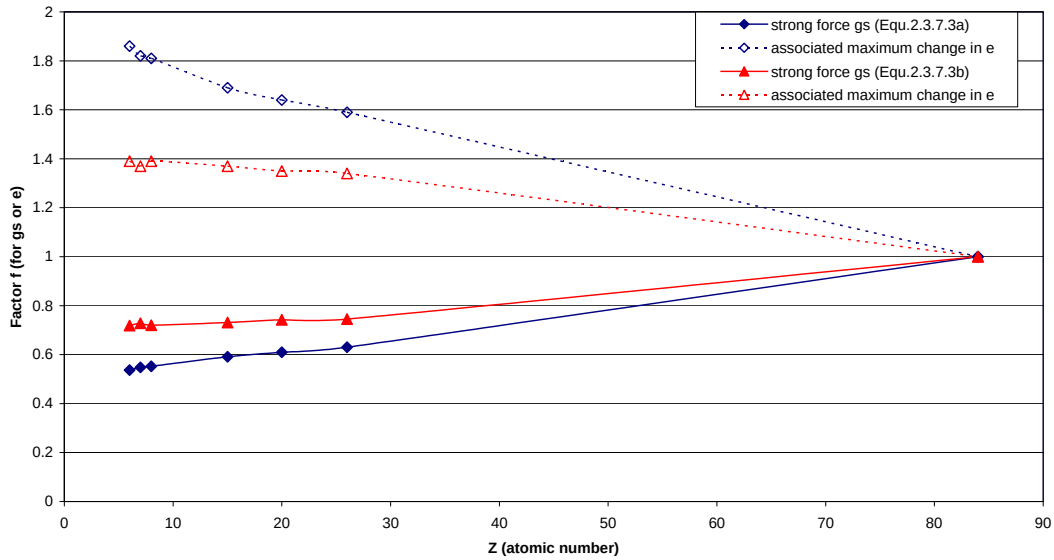
- The interpretation is that
  EITHER $\lambda_s = 1$ and $\lambda_{em} = \lambda > 1$ (first two columns – blue)
  OR $\lambda_{em} = 1$ and $\lambda_s = 1/\lambda < 1$ (the other columns - black)
- Consequently the alternative Equ.(2.3.7.2b): $\lambda_{em} = \frac{g_s^2}{g_{s0}^2} \cdot \frac{\alpha}{\alpha_0}$ is relevant in the 'other' columns, and yields the values which differ from unity.
- [(1)]Using (2.3.7.3a); [(2)]Using (2.3.7.3b).
- The electrostatic force strength is indicated by the (dimensionless) quantum of charge, $e = \sqrt{\alpha}$, since this is the relevant coupling constant comparable with g_s.

The results from the above Table are plotted against Z in the following two graphs. The first corresponds to the interpretation $\lambda_s = 1$ and $\lambda = \lambda_{em}$ (the blue data from the Table). This graph therefore only shows the change in e, there is no change in g_s. The second graph is for the interpretation $\lambda_{em} = 1$ and $\lambda_s = 1/\lambda$. This graph shows the change in g_s, as well as the maximum required simultaneous change in e.

Electric Charge Quantum e Varied by Factor f (Nuclear Force Unchanged)



Nucear Stability Against Fission: Strong Nuclear Force Factored By f, Electromagnetic Binding Energy Unchanged



Stability Against Weak Decays

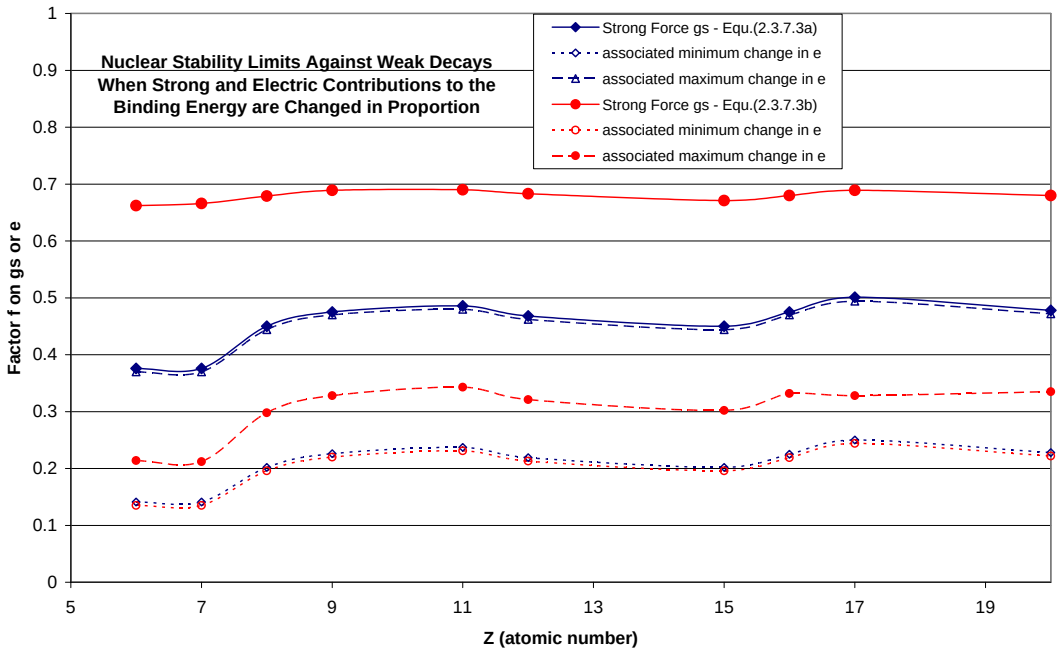
Results of Varying the Strong Nuclear and Electric Energies in Proportion

Element	$\lambda_s = \lambda_{em} = \lambda$	g_s / g_{s0} (1)	e / e_0 (1)	g_s / g_{s0} (2)	e / e_0 (2)
C(6)	0.02	0.376	0.141 to 0.376	0.662	0.141 to 0.214
N(7)	0.02	0.376	0.141 to 0.376	0.666	0.141 to 0.212
O(8)	0.041	0.450	0.202 to 0.450	0.679	0.202 to 0.298
F(9)	0.051	0.475	0.226 to 0.475	0.689	0.226 to 0.328
Na(11)	0.056	0.486	0.237 to 0.486	0.690	0.237 to 0.343
Mg(12)	0.048	0.468	0.219 to 0.468	0.683	0.219 to 0.321
P(15)	0.041	0.450	0.202 to 0.450	0.671	0.202 to 0.302
S(16)	0.051	0.475	0.225 to 0.475	0.680	0.225 to 0.332
Cl(17)	0.063	0.501	0.250 to 0.501	0.689	0.250 to 0.328
Ca(20)	0.052	0.478	0.228 to 0.478	0.680	0.228 to 0.335

Notes:-

- The interpretation is that $\lambda_s = \lambda_{em} = \lambda$
- The two different values for e/e_0 are from Equ.(2.3.7.2a) and Equ.(2.3.7.2b) respectively. The former is simply that $\alpha/\alpha_0 = \lambda$. The alternative is $\lambda = \frac{g_s^2}{g_{s0}^2} \cdot \frac{\alpha}{\alpha_0}$.
- (1)Using (2.3.7.3a); (2)Using (2.3.7.3b):

These results are shown graphically below:-



Stability Against Weak Decays

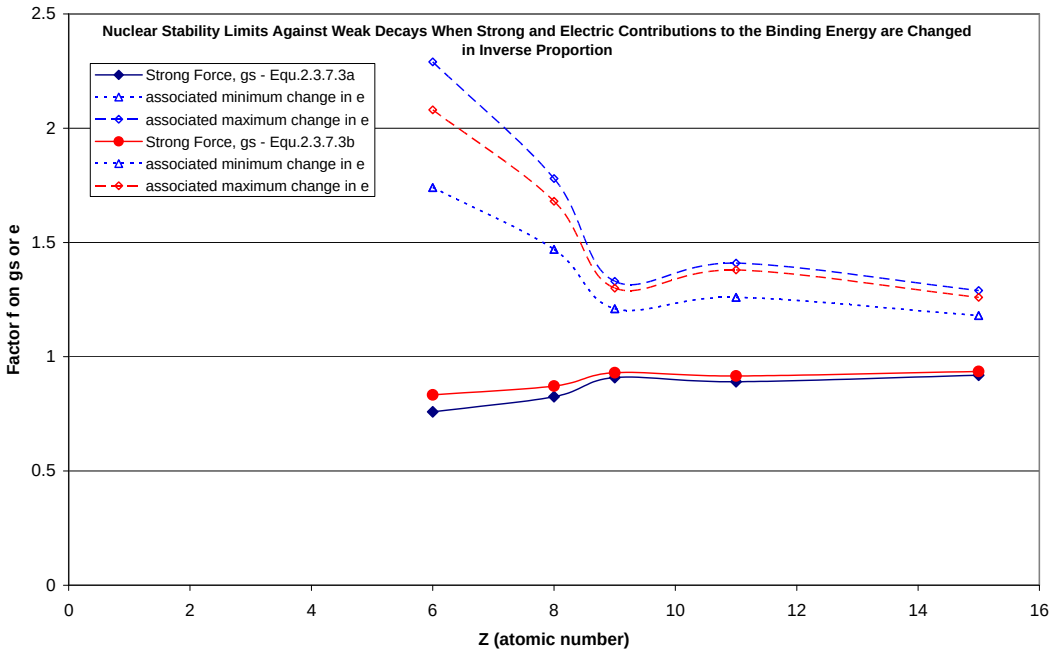
Varying the Strong Nuclear and Electric Energies in Inverse Proportion

Element	$\lambda_s = 1/\lambda_{em} = \lambda$	g_s / g_{s0} (1)	e/e_0 (1)	g_s / g_{s0} (2)	e/e_0 (2)
C(6)	0.332	0.759	1.74 to 2.29	0.833	1.74 to 2.08
O(8)	0.464	0.825	1.47 to 1.78	0.872	1.47 to 1.68
F(9)	0.683	0.909	1.21 to 1.33	0.930	1.21 to 1.30
Na(11)	0.630	0.891	1.26 to 1.41	0.916	1.26 to 1.38
P(15)	0.714	0.919	1.18 to 1.29	0.936	1.18 to 1.26

Notes:-

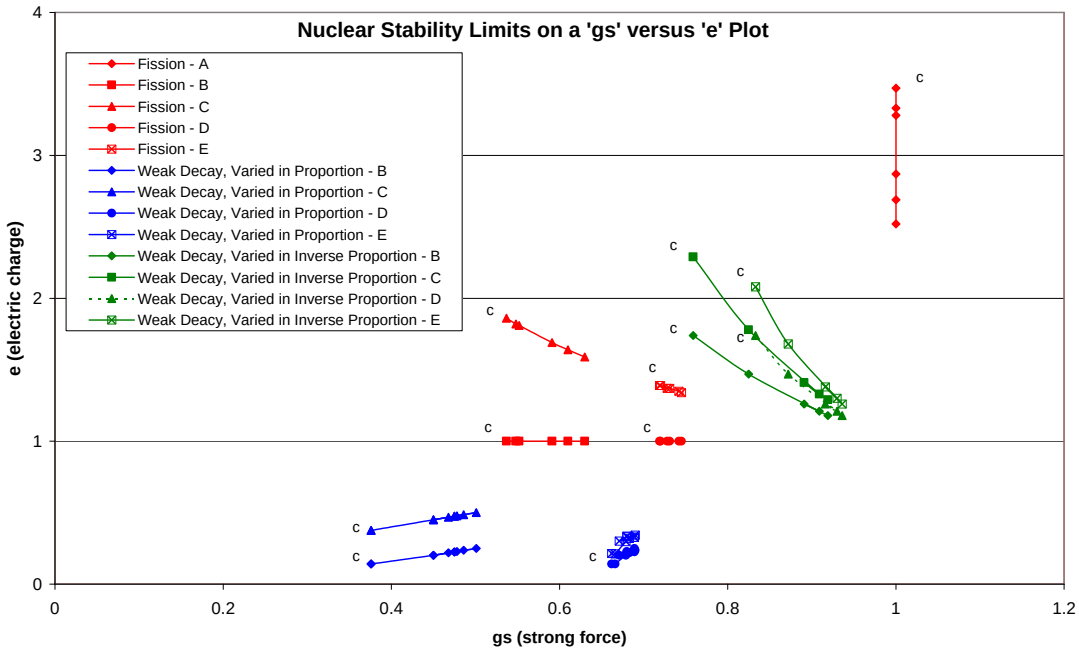
- The interpretation is that $\lambda_s = 1/\lambda_{em} = \lambda$
- The two different values for e/e_0 are from Equ.(2.3.7.2a) and Equ.(2.3.7.2b) respectively. The former is simply that $\alpha/\alpha_0 = \lambda$. The alternative is $\lambda = \frac{g_s^2}{g_{s0}^2} \cdot \frac{\alpha}{\alpha_0}$.
- (1)Using (2.3.7.3a); (2)Using (2.3.7.3b):

These results are shown graphically below:-



This gives a much tighter lower bound on gs compared with varying both forces in proportion. This is because the simultaneous increase in the strength of the electric force promotes instability and prohibits too great a decrease in gs whilst remaining stable. In fact, this lower bound on gs is more restrictive than that derived above for stability against fission / alpha decay.

Finally, in the next graph, we plot all the nuclear stability limits derived above in (gs, e) space. On this plot, nuclear stability obtains for (gs, e) values which lie along a vector pointing towards (1, 1) from the plotted points. Conversely, nuclear instability is predicted for (gs, e) values which lie along a vector pointing away from (1,1) from the plotted points.



The lines indicate the different bounds derived for different elements (i.e. different Z). The end at which carbon occurs is indicated – it lies furthest away from (1, 1), i.e. the heavier elements are more sensitive to changes in the force strengths. The key is as follows:-

B gs from Equ.2.3.7.3a; e from Equ.2.3.7.2a
C gs from Equ.2.3.7.3a; e from Equ.2.3.7.2b
D gs from Equ.2.3.7.3b; e from Equ.2.3.7.2a
E gs from Equ.2.3.7.3b; e from Equ.2.3.7.2b

4. Summary and Conclusions

In order for complex structures such as life to exist, it is necessary for the building blocks of matter – the atoms – to be capable of binding together. For such complex structures as life to exist for the lengths of time necessary for Darwinian evolution to function, atoms and molecules must be very stable. In this Chapter the conditions leading to the stability of the nuclei of atoms have been explained. Chapter 10 will address the stability of atoms themselves, that is the structures formed of nuclei and orbiting electrons. Chapter 11 considers the stability of molecules: the bound states of many atoms.

The discussion of the stability of nuclei uses an approximation for their binding energy known as the liquid drop model. The predictions of the model are tested against the known stability or instability of nuclei in this universe. This covers both fission and weak-force mediated radioactive decays. Having gained confidence that the model is a reasonably indicative guide, the implications of varying the strength of the strong nuclear force (gs) and the electrostatic force (e) are assessed.

As regards fissile instability, the criteria usually adopted are: (a)an increase in total binding energy, or, (b)instability with respect to small shape changes. These are considered but found to be little or no more restrictive than a much simpler criterion, namely that the binding energy be greater than zero. Whichever criterion is employed, instability against fission results from an increase in the *relative* strength of the electrostatic force compared with the

strong-nuclear force. Changing the strength of both these forces equally would not lead to fissile instability. (NB: In this context, "force" strictly means "contribution to the binding energy"). The factor by which the ratio of the electrostatic force to strong-nuclear force must be increased to make nuclei unstable is found as a function of atomic number, Z.

In the case of carbon, and assuming no change in the electrostatic energy, fissile instability occurs if gs is reduced by a factor of between 0.54 and 0.72, depending upon the assumed dependence of the binding energy on gs. If the nuclear force is unchanged, nuclear instability occurs in carbon if e is increased by a factor of 3.5. Heavier elements are more sensitive to changes in the strength of the forces, becoming unstable for smaller percentage changes. The sensitivity of nuclear stability to gs is exaggerated by the fact that the binding energy varies as gs^4, or perhaps more sensitively still. Thus, the lower bound for gs of 0.54-0.72 for carbon results from a change in the relative electrostatic:nuclear binding energies of a factor of ~12.

Stability against the weak-force mediated decay modes, beta decay or electron capture, can be expressed in terms of the binding energy difference of nuclei with Z differing by 1. Specifically, beta decay is not possible if B(Z)-B(Z+1) exceeds the mass deficit Mn – Mp – me, whereas electron capture is not possible if B(Z-1)-B(Z) is less than this mass deficit. Decays by positron emission are also subsumed in these criteria. Thus, for example, weak decay modes can be induced by reducing sufficiently the binding energy contributions of both the strong-nuclear force and the electrostatic force in proportion. To induce weak decay instability in carbon, both these binding energy contributions must be reduced by a factor of 0.02. This corresponds to a reduction in gs by a factor of between 0.38 and 0.66, with a simultaneous reduction in e by a factor of between 0.14 and 0.38. The corresponding results are presented for other biologically important elements.

Weak decays induced by non-proportional changes in the force strengths have also been considered. Specifically we assume that their binding energy contributions vary in inverse proportion. This leads to a more restrictive lower bound on gs than the previous cases. For carbon, weak decay modes occur if the nuclear force component of the binding energy is reduced by x0.333, whilst the electrostatic energy is increased byx3.33. This corresponds to a reduction in gs by a factor of between 0.76 and 0.83, with a simultaneous increase in e by a factor of between 1.74 and 2.3. Heavier elements are again found to be more sensitive to changes in the strength of the forces, becoming unstable for smaller percentage changes.

These results suggest that nuclear stability requires a Type C1, single-sided 'coincidence' in gs, and also a Type D1, single-sided 'coincidence' in e. However, the sensitivity of nuclear stability to gs is exaggerated by the fact that the binding energy varies as gs^4, or perhaps more sensitively still. In terms of binding energies, both 'coincidences' would be classed as Type D1, single-sided.