Chapter 9CL – The Effect of Diproton Stability on the Universe

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1. Introduction

Having determined the pp capture cross section in Appendix 4 of the Tutorial we are now in a position to determine whether the pp capture reaction rate is sufficiently fast to affect, (a)the outcome of Big Bang nucleosynthesis in the first few minutes, and, (b)the energy production rates and hence lifetimes of stars. We shall find that Big Bang nucleosynthesis is unaffected whereas stars are greatly affected. Contrary to the claims that have generally been made in the literature, however, it does appear that stable stars could form. They would be quite different from the stars of this universe, but would be capable of forging the chemical elements, and be sufficiently hot and luminous to nurture life.

In Appendix A4 of the Tutorial we showed that the pp capture rate is much slower than the np capture rate as a result of pp capture involving identical particles. In this Chapter we show that the pp capture rate is sufficiently slow that no significant numbers of diprotons are formed during Big Bang nucleosynthesis, and the stability of the diproton is irrelevant. Only when stars form does the stability of the diproton make a difference to the universe. This is also contrary to the impression given in the literature, where the scenario depicted is that diproton stability would lead to an all-helium primordial universe, forever devoid of hydrogen and hence incapable of forming water, hydrocarbons, proteins, or anything with a hydrogen bond. Life based on anything remotely like familiar biochemistry would not be possible in such a universe – but this is not the true outcome of diproton stability.

In this Chapter we use the single-energy cross sections of Appendix A4 to find the pp reaction rate at a given temperature. The latter involves the usual integral over the product of the tail of the Maxwell distribution and the energy-sensitive cross-section (i.e. the Gamow peak). To estimate the Gamow peak we use a simple analytic approximation for the cross-section based on the cross section with no Coulomb barrier times a Coulomb barrier factor. This rough estimate is then subject to a simple 'empirical' correction factor by comparing it to the numerical solutions of the Tutorial Appendix A4.

A reaction is frozen out by cosmic expansion when its rate drops below the Hubble parameter. Thus we find the time (temperature) for freeze-out of diproton formation. We can also estimate the earliest time at which diprotons become stable to photodisintegration, simply by equating the number of photons with energy sufficient to cause photodisintegration with the maximum possible number of diprotons. We find that the diproton formation reaction is frozen-out before diprotons become stable. Consequently, no significant numbers of diprotons survive period of Big Bang nucleosynthesis.

It is also shown that no significant numbers of dineutrons would be produced either.

The second half of this Chapter investigates how stars might behave in a universe with a stable diproton. This changes the initial nuclear reaction in stars in a crucial way, because rather than the weak-interaction mediated $p + p \to D + e^+ + \nu_e$, which is extremely slow, we have the electro-strong reaction $p + p \rightarrow^2_2 He + \gamma$ followed by the weak decay $^2_2 He \rightarrow^2_1 D + e^+ + \nu_e$, which together are roughly 10 orders of magnitude faster at the same temperature and density. It is entirely false to conclude that stars would, as a consequence, be explosively unstable. The reason is very simple: the temperature and density are *not* the same. Nuclear reaction rates are so very sensitive to temperature (exponentially so) that a relatively modest reduction in temperature can bridge this enormous rate difference and produce stars which are not so very different from those in our universe.

This is demonstrated here by constructing an explicit example. We cannot claim that this is conclusive. To do so a detailed stellar model including all the relevant opacities, heat transfer mechanisms, spatial variations of the field variable, etc, would be required. However, by considering the more elementary requirements for stellar stability the claim is made plausible.

2. Data Assumptions

The data assumed are consistent with the Tutorial Appendix A4 in which the cross section for pp formation is derived. We shall consider the strong nuclear coupling constant (g_s) to be increased in strength by 20%, 30% or 40%, corresponding to increases in the potential well depth of x1.44, x1.69 or x1.96. In Appendix A4 we found that, for energies below 1 MeV, the cross section is largest for a 30% increase in g_s. Hence, reaction rates are evaluated for this case only. The assumed singlet nuclear potential is thus V_0 = 27.209 MeV with a range of radius a = 2.4 fm (see the Tutorial, Appendices A2 and A4).

3. Mean CoM Energy and Relative Speed For A Thermal Maxwell Distribution

Both in the minutes after the Big Bang and at the centre of stars the protons are in thermal equilibrium with a Maxwell distribution of speeds (or energies). This can be written,

$$P[\tilde{v}]d\tilde{v} = \frac{4}{\sqrt{\pi}}\tilde{v}^2 e^{-\tilde{v}^2} d\tilde{v} \qquad \text{where} \quad \tilde{v} = \frac{v}{v_0}, \quad v_0 = \sqrt{\frac{2kT}{M_p}} \qquad (1)$$

If the velocities of a pair of protons are $\overline{v}_1$ and $\overline{v}_2$ then the centre of mass moves at velocity $(\overline{v}_1 + \overline{v}_2)/2$ and the velocity of each proton in the CoM system is $\pm(\overline{v}_1 - \overline{v}_2)/2$. Hence, the sum of the kinetic energies of the two protons in the CoM system is,

$$E = 2 \text{ x } \frac{1}{2} M_p \left|(\overline{v}_1 - \overline{v}_2)/2\right|^2 = \frac{1}{4} M_p v_r^2 \qquad (2)$$

where,

$$v_r^2 = \left|\overline{v}_1 - \overline{v}_2\right|^2 = v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta \qquad (3)$$

and v_r is the relative speed of the two incident protons, and θ is the angle between their initial velocities. Since both protons are taken from a Maxwell distribution, the mean relative speed is thus,

$$<v_r>=\frac{1}{2}\int v_r \cdot P[\tilde{v}_1]d\tilde{v}_1 P[\tilde{v}_2]d\tilde{v}_2 d(\cos\theta) \quad (4)$$

and the mean of the speed-squared is,

$$<v_r^2>=\frac{1}{2}\int v_r^2 \cdot P[\tilde{v}_1]d\tilde{v}_1 P[\tilde{v}_2]d\tilde{v}_2 d(\cos\theta) \quad (5)$$

and so the mean CoM energy, from (2), is,

$$<E>=\frac{1}{4}M_p <v_r^2> \quad (6)$$

Numerical integration of the above gives,

$$<v_r>=1.59588 v_0 \quad \text{and} \quad <v_r^2>=3v_0^2 \quad (7)$$

The mean speed of a Maxwell distribution is,

$$<v>=1.128353 v_0 \quad (8)$$

so we see that, $<v_r>=\sqrt{2}<v>$ (9)

Substituting the second of (7) into (6) gives, $<E>=\frac{3}{4}M_p v_0^2=\frac{3}{2}kT$ (10)

Note that the mean energy of a single proton is also this value, 3kT/2. Thus the total CoM energy of the two protons is the same as that of one proton in the 'lab' frame.

The import of these observations is that, for a best estimate calculation of reaction rates, we shall assume for a given temperature that the mean CoM energy is given by (10) and the mean relative speed given by (7).

4. The Temperature and Proton Density

To a sufficient accuracy for our purposes we will assume time and temperature to be related by $T=10^{10}/\sqrt{t}$ (K and seconds). Strictly this applies only after electron-positron annihilation, i.e. after about 14 seconds.

The proton density is estimated assuming all neutrons have already combined to form deuterons. In this universe, this happens after roughly 3 minutes. We shall be considering diproton formation at times earlier than this. However, this is brought about by postulating an increased strength of nuclear force, in which case the binding energy of the deuteron is also larger and deuterons will be stable at earlier times. Hence it is consistent to assume that most, if not all, neutrons have been mopped up as deuterons before diproton formation begins. Thus we shall use a proton number density of,

$$\rho_p^N=0.75\frac{\rho_\gamma^N}{\xi}, \qquad \rho_\gamma^N=0.2436\left(\frac{kT}{\hbar c}\right)^3, \quad \xi=1.9\text{x}10^9 \quad (11)$$

5. Times of Stability

At sufficiently early times the temperature of the universe is so high that any diprotons formed will immediately photodisintegrate. In the Tutorial Chapter 6 we introduced a method for estimating the temperature, and hence time, at which a bound state would be unstable against photodisintegration. The method is simply to equate the number of photons with energies greater than the binding energy of the composite particle with the maximum possible number of these particles. Now the maximum possible number of diprotons is half the initial number of protons, given by (11), above. Thus, the maximum possible number of diprotons per photon is $0.375/\xi \approx 2 \times 10^{-10}$. From the Tutorial Chapter 6, Equs.(20, 22), the number of photons with energy greater than B is,

$$0.416\int_{x_1}^{\infty}\frac{x^2dx}{e^x-1} = 0.416\left[2+2x_1+x_1^2\right]e^{-x_1} = 2 \times 10^{-10} \qquad (12)$$

where, $x_1 = B/kT$. The solution of (12) is $x_1 = 28.22$. Thus, for the range of binding energies of the diproton considered we find the earliest times for stability to be,

g / g_{actual}	1.2	1.3	1.4
B (MeV)	0.59178	1.99683	4.02317
T (K)	2.4×10^8	8.2×10^8	1.65×10^9
t (sec)	1,700	150	37

Assuming the initial number of neutrons (0.84×10^{-10} per photon), and hence $x_1 = 29.145$, the times of stability of the dineutron are,

g / g_{actual}	1.2	1.3	1.4
B (MeV)	0.81592	2.21463	4.2215
T (K)	3.25×10^8	8.81×10^8	1.68×10^9
t (sec)	950	130	35

The first of these ($g/g_{actual} = 1.2$) indicates dineutron stability well after the deuteron becomes stable (in *this* universe, i.e. ~ 3 minutes). Consequently, it is clear that deuteron formation (and subsequent helium formation) will mop up all the neutrons before any significant numbers of dineutrons have chance to form stablely. We may surmise that this will also be the case for the greater increases in g, but we need to estimate the increased deuteron binding energy to confirm this. Chapter 9B gives an estimation formula,

$$\frac{B}{B_{actual}} = \left(\frac{g/g_{actual} - 0.85}{1-0.85}\right)^2 \qquad (13)$$

Thus, for g/g_{actual} of 1.3 and 1.4 the deuteron binding energies are estimated to be 9 MeV and 13.4 MeV respectively. These correspond to deuteron stability times of ~8 seconds and ~3 seconds. Consequently, there is no question of significant numbers of dineutrons forming in these cases either. The free neutrons will have disappeared before dineutrons become stable.

6. pp Capture: Reaction Times and Freeze-Out

We have seen in the Tutorial that if a reaction time exceeds the reciprocal of the Hubble parameter, the reaction is frozen out by cosmic expansion. We shall see that this is indeed the case for diproton formation. In fact, we shall see that the reaction is frozen out even before the diproton becomes stable – and hence no significant numbers of diprotons are formed. The reaction will remain frozen out so long as the universe continues to be homogeneous, i.e. until the first proto-stars or proto-galaxies form. Note that, during the radiation dominated era, the reciprocal of the Hubble parameter is twice the age of the universe. Thus, the condition for freeze-out is that the reaction time exceed 2t.

We must first evaluate the pp capture reaction rate for a thermal distribution of proton energies. This is done following the method of Tutorial Chapter 14. Since the cross-section has a different energy dependence from that of the electric dipole interaction considered in Chapter 14 we shall carry out the derivation in full. We first require an analytic approximation for the reaction rate at a specified energy. The starting point is the value the pp capture cross-section would take if the Coulomb barrier were switched off (for an electric quadrupole interaction). This is given in the Tutorial Appendix A4 as,

$$\sigma_{cap}^{E} = \frac{64\pi\alpha}{15} \cdot (\hbar c)^2 \cdot \frac{B^{1/2} E^{3/2}}{(M_p c^2)^3 (E+B)} \qquad (6.1)$$

We now need to account for the Coulomb barrier, this being identical to Chapter 14 and the Annex to Tutorial Chapter 18:-

The energy dependence of the matrix element derives from how much of the free state wavefunction manages to "penetrate the Coulomb barrier" from outside. It is the magnitude of the free wavefunction within the region of the Coulomb potential which is significant, (i.e. radii 'r' such that, $a < r < r_E = Z_1 Z_2 \tilde{e}^2 / E$). This is the region within which the free state has negative kinetic energy, the exponential decay length being the reciprocal of $\kappa = \sqrt{2m(V(r) - E)} / \hbar$, noting that this varies with 'r', since $V(r) = Z_1 Z_2 \tilde{e}^2 / r$.

Roughly speaking, we may expect the magnitude of the wavefunction to diminish from $r = r_E$ to $r = a$ by a factor $\exp\left\{ -\int_a^{r_E} \kappa dr \right\}$. The integral may be evaluated exactly to give a factor,

$$X = \exp\left\{ -\frac{Z_1 Z_2 \tilde{e}^2 \sqrt{2m_R}}{\hbar} \cdot \frac{1}{\sqrt{E}} \left[\theta - \frac{1}{2}\sin 2\theta \right] \right\} \quad \text{where, } \theta = \tan^{-1}\sqrt{\frac{V_c}{E} - 1}, \qquad (6.2)$$

where, $V_c = Z_1 Z_2 \frac{\tilde{e}^2}{a}$ and m_R is the reduced mass, i.e. $m_R = \frac{A_1 A_2}{A_1 + A_2} M_p$. (6.3)

For energies sufficiently small compared with V_c, $\theta \to \pi/2$ and hence the sin term is zero and we get,

$$X \to \exp\left\{-b/2\sqrt{E}\right\} \quad \text{where,} \quad \frac{b}{2} = \frac{\pi Z_1 Z_2}{2}\alpha\sqrt{\frac{2A_1A_2}{A_1+A_2}M_p c^2} \tag{6.4}$$

[Noting that we have changed our convention from the Tutorial Chapter 18 and Appendix A3, calling the expression in (6.4) b/2 rather than b]. In the case of interest the two incident particles have $Z_1 = Z_2 = A_1 = A_2 = 1$, and so b / 2 = 0.351 √MeV.

Hence, recalling that the cross section depends on the square of the matrix element, our first (crude) analytic approximation for the pp capture cross section is,

$$\sigma_{cap}^{E} = \frac{64\pi\alpha}{15}\cdot(\hbar c)^2\cdot\frac{B^{1/2}E^{3/2}}{(M_p c^2)^3(E+B)}e^{-b/\sqrt{E}} \tag{6.5}$$

We will see later that this is rather too crude an approximation, but it suffices for an initial attempt. As written, (6.5) assumes that the Coulomb barrier only becomes irrelevant for energies >> b^2. Actually this is comparable with the height of the Coulomb barrier, and hence it is more accurate to write,

$$\sigma_{cap}^{E} = \frac{64\pi\alpha}{15}\cdot(\hbar c)^2\cdot\frac{B^{1/2}E^{3/2}}{(M_p c^2)^3(E+B)}e^{-b\left(\frac{1}{\sqrt{E}}-\frac{1}{\sqrt{E_0}}\right)} \tag{6.6}$$

where E_0 is roughly 0.5 MeV or so. Equs.(6.5, 6.6) differ from the equivalent for the electric dipole interaction of Chapter 14 in respect of the numerator's dependence upon $E^{3/2}$, rather than $E^{1/2}$ as for the dipole interaction.

The reaction rate per proton is $\rho_N^p v \sigma_{cap}^E$, where the first term is the number density of protons, and v is the relative closing speed of the incident protons, i.e.

$$v = 2\hbar k / M_p = 2\sqrt{E/M_p} \tag{6.7}$$

assuming non-relativistic speeds. (Recall that E is the sum of the two protons' kinetic energies in the CoM frame). Assuming the proton density is 1 mole per cm^3, i.e. A per m^3, where A = 6.02 x 10^{29}, then the reaction rate per proton is,

$$\begin{aligned}\text{Reaction Rate} &= \frac{128\pi}{15}A\alpha\left(\frac{\hbar}{M_p c}\right)^2\frac{E^2}{M_p c^2(E+B)}\sqrt{\frac{B}{M_p}}e^{-b\left[\frac{1}{\sqrt{E}}-\frac{1}{\sqrt{E_0}}\right]} \\ &\approx \frac{128\pi}{15}\tilde{A}\alpha\left(\frac{\hbar}{M_p c}\right)^2\frac{E^2}{M_p c^2 B}\sqrt{\frac{B}{M_p}}e^{-b/\sqrt{E}}\end{aligned} \tag{6.8}$$

where the last expression assumes E << B, which is a good approximation in our applications, and where $\tilde{A} = Ae^{b/\sqrt{E_0}}$. The weighted average of (6.8) over the Maxwell distribution of energies is considered next.

The derivation is the same as that already considered in Chapter 14 except that we note that (6.8) has a quadratic energy dependence rather than a linear dependence (in addition, of course, to the dominant exponential dependence). For completeness we include the derivation in full:-

For the non-relativistic energies of interest, the kinetic energies of the reacting particles are given by the Maxwell distribution,

$$P[\varepsilon]d\varepsilon = \frac{2}{\sqrt{\pi}}\sqrt{\varepsilon}e^{-\varepsilon}d\varepsilon \tag{6.9}$$

where $\varepsilon = E/kT$. We denote the reaction rate at temperature T by R[T]. If one proton has speed v_1 in the lab frame, and the other speed v_2, and if the two velocities were at an angle θ, the resulting reaction rate is denoted $R[v_1, v_2, \theta]$. If we know the latter we can find R[T] from,

$$R[T] = \int_0^\infty P[\varepsilon_1]d\varepsilon_1 \int_0^\infty P[\varepsilon_2]d\varepsilon_2 \int_0^\pi \frac{d(\cos\theta)}{2} \cdot R[v_1, v_2, \theta] \tag{6.10}$$

where, $\varepsilon_{1,2} = \dfrac{M_p v_{1,2}^2}{2kT}$. Because the random motions are isotropic, all solid angles are equally likely, hence the uniform probability density in cosθ. Now $R[v_1, v_2, \theta]$ will be given by Equ.(6.8) provided that we find the CoM system total kinetic energy (E) in terms of v_1, v_2 and θ. Noting that for non-relativistic speeds the relative speed of the two protons is the same in the lab and CoM frames, the cosine theorem gives,

$$E = \frac{1}{2}(E_1 + E_2) + \frac{Mv_1v_2}{2}\cos\theta \tag{6.10a}$$

By averaging over angles the second term disappears, i.e.,

$$\langle E \rangle = \int_0^\pi E\frac{d(\cos\theta)}{2} = \frac{1}{2}(E_1 + E_2) \tag{6.10b}$$

Similarly, $$\langle E^2 \rangle = \int_0^\pi E^2 \frac{d(\cos\theta)}{2} = \frac{1}{4}(E_1 + E_2)^2 + \frac{1}{3}E_1E_2 \approx \frac{1}{3}(E_1 + E_2)^2 \tag{6.10c}$$

where the last form is a very crude approximation (valid at the maximum value, when $E_1 = E_2$). It will be used to approximate the E^2 factor in (6.8).

These approximations are not appropriate for use inside the exponential factor, $e^{-b/\sqrt{E}}$ in (6.8) however. This is because the dominant contribution to the integral will come from the angle which results in the largest E for fixed E_1 and E_2. From (6.10a) this is clearly $\theta = 0$. (6.10a) then reduces to,

$$E = \frac{1}{2}(E_1 + E_2) + \frac{Mv_1v_2}{2} = \frac{1}{2}(E_1 + E_2) + \sqrt{E_1E_2} \approx (E_1 + E_2) \quad (6.10d)$$

where the last form is again the very crude approximation which is valid at the maximum value, i.e. when $E_1 = E_2$. This is the approximation which will be used within the exponent. We can now derive an approximate analytic expression for the temperature dependence of the reaction rate from (6.8) and (6.10), using the approximations (6.10c) and (6.10d), i.e.,

$$R[T] = \frac{4}{\pi} R_0 \int_0^\infty \int_0^\infty d\varepsilon_1 d\varepsilon_2 \sqrt{\varepsilon_1\varepsilon_2} \frac{(\varepsilon_1 + \varepsilon_2)^2}{3} \exp - \left\{ \frac{\tilde{b}}{\sqrt{\varepsilon_1 + \varepsilon_2}} + \varepsilon_1 + \varepsilon_2 \right\} \quad (6.11)$$

where, $$\tilde{b} = b / \sqrt{kT} \quad (6.12)$$

and, $$R_0 = \frac{128\pi}{15} \tilde{A}\alpha \left(\frac{\hbar}{M_p c} \right)^2 \frac{(kT)^2}{M_p c^2 B} \sqrt{\frac{B}{M_p}} \quad (6.13)$$

and where we have used the dimensionless variables,

$$\varepsilon_1 = E_1 / kT, \ \varepsilon_2 = E_2 / kT, \ \varepsilon = \varepsilon_1 + \varepsilon_2 \quad (6.14)$$

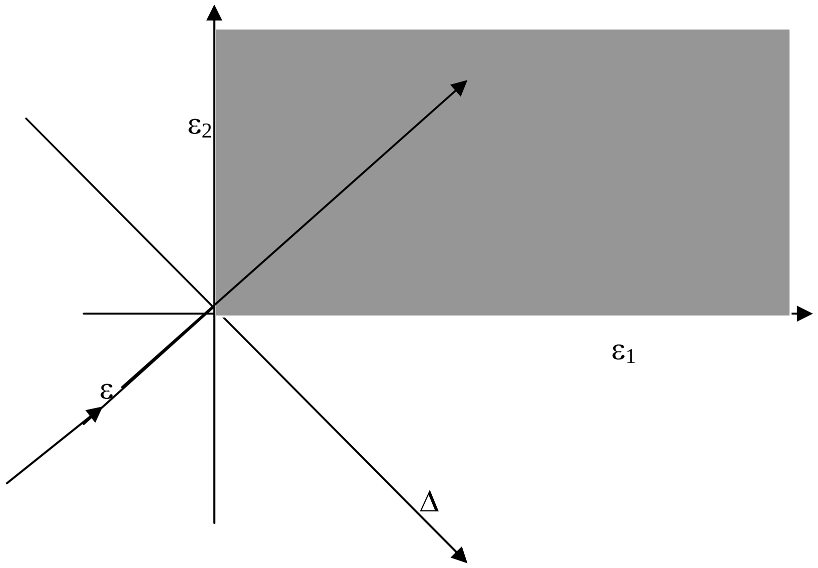
We now change the integration variable to ε and $\Delta = \varepsilon_1 - \varepsilon_2$, noting that,

$$\varepsilon_1 = (\varepsilon + \Delta)/2 \ \text{ and } \ \varepsilon_2 = (\varepsilon - \Delta)/2 \quad (6.15)$$

so that the Jacobean ($d\varepsilon_1 d\varepsilon_2 / d\varepsilon d\Delta$) takes the value ½. Hence we have,

$$R[T] = \frac{R_0}{3\pi} \int_0^\infty \varepsilon^2 \cdot \exp - \left\{ \frac{\tilde{b}}{\sqrt{\varepsilon}} + \varepsilon \right\} \cdot d\varepsilon \int_{-\varepsilon}^{+\varepsilon} \sqrt{\varepsilon^2 - \Delta^2} \cdot d\Delta \quad (6.16)$$

Note that the range of integration $\varepsilon_1 \in [0, \infty]$ and $\varepsilon_2 \in [0, \infty]$ maps into $\varepsilon \in [0, \infty]$ and $\Delta \in [-\varepsilon, +\varepsilon]$, as may be seen from the following (where the region of integration is shaded)…



The second integral is just $\pi\varepsilon^2/2$, so (19) becomes,

$$R[T] = \frac{R_0}{6}\int_0^\infty \varepsilon^4 d\varepsilon \cdot \exp - \left\{ \frac{\tilde{b}}{\sqrt{\varepsilon}} + \varepsilon \right\} \tag{6.17}$$

Equ.(6.17) differs from the equivalent in Tutorial Chapter 14 in having a factor of ε^4 rather than ε^3 in the integrand. This stems from the quadratic rather than linear energy dependence on energy of the reaction rate (which in turn is due to pp capture being a quadrupole interaction rather than a dipole interaction). Calling the exponent,

$$f(\varepsilon) = \frac{\tilde{b}}{\sqrt{\varepsilon}} + \varepsilon \tag{6.18}$$

we note that it attains a minimum when,

$$\varepsilon_0 = (0.5\tilde{b})^{2/3} = \frac{(0.5b)^{2/3}}{(kT)^{1/3}} = E_0 / kT \Rightarrow E_0 = (0.5bkT)^{2/3} \tag{6.19}$$

The minimum value of the exponent is,

$$f_{min} = 3\left(\frac{\tilde{b}}{2}\right)^{2/3} \tag{6.20}$$

The minimum in f is due to the competition between the energy dependence of the matrix element and the Maxwell distribution. At a given temperature, the Maxwell distribution means that there are far more protons with energies around kT than with much higher energies. However, for the lower energy protons, the matrix element is much smaller, and hence the reaction rate much slower, due to the greater difficulty low energy protons have in penetrating the Coulomb barrier. Conversely, at the same temperature, the Maxwell distribution ensures that there are fewer protons with energies sufficiently great to penetrate the Coulomb barrier easily and hence to result in a faster reaction rate

We now find an approximate closed-form expression for the total reaction rate, carrying out the integral in (6.20) by expanding the exponent as a power series, i.e.,

$$f(\varepsilon) \approx f_{min} + 0.5 f_0''(\varepsilon - \varepsilon_0)^2 + \ldots.. \tag{6.21}$$

where,

$$f_0'' = \frac{3}{2^{1/3}\tilde{b}^{2/3}} \tag{6.22}$$

Hence, (6.17) gives,

$$\begin{aligned} R[T] &\approx \frac{R_0}{6}\int_0^\infty \varepsilon^4 d\varepsilon \cdot \exp - \left\{f_{min} + 0.5 f_0''(\varepsilon-\varepsilon_0)^2\right\} \\ &\approx \frac{R_0}{6}\varepsilon_o^4 e^{-f_{min}} \int_0^\infty d\varepsilon \cdot \exp - \left\{0.5 f_0''(\varepsilon-\varepsilon_0)^2\right\} \\ &= \frac{R_0}{6}\varepsilon_o^4 e^{-f_{min}} \int_{-\varepsilon_0}^\infty dy \cdot \exp - \left\{0.5 f_0'' y^2\right\} \\ &\approx \frac{R_0}{6}\varepsilon_o^4 e^{-f_{min}} \int_{-\infty}^\infty dy \cdot \exp - \left\{0.5 f_0'' y^2\right\} \\ &= \frac{R_0}{6}\varepsilon_o^4 e^{-f_{min}} \sqrt{\frac{2\pi}{f_0''}} \end{aligned} \tag{6.23}$$

The derivation of (6.23) requires that $\varepsilon_0 >> 1$, which is equivalent to the requirement that the temperature is less than 10^9 K, and hence is a good approximation for stars. Substituting (6.19) and (6.22) into (6.23) gives,

$$\begin{aligned} R[T] &\approx \frac{1}{3}\sqrt{\frac{\pi}{3}} R_0 \left(\frac{\tilde{b}}{2}\right)^3 \exp\{-f_{min}\} \\ &\approx \frac{16}{45}\pi b^3 \alpha \tilde{A}\left(\frac{\hbar}{M_p c}\right)^2 \left(\frac{1}{M_p c^2 \sqrt{BM_p}}\right) \cdot \sqrt{kT}\exp\{-f_{min}\} \end{aligned} \tag{6.24}$$

where f_{min} is given by (22) and (14) and b = 0.7024 √MeV. Note that the temperature dependence of the reaction rate is given predominantly by the exponential factor, since √kT varies relatively slowly.

The binding energy, B, of the diproton has been given in Section 5, i.e. about 0.6, 2.0 or 4.0 MeV for increases in g_s of x1.2, x1.3, x1.4 respectively. [NB: There is no singularity in the reaction rate as B → 0 really, since the term B in the denominator is an approximation for (B + E)]. For sake of argument, using B = 2MeV (i.e. for g_s increased by x1.3) and considering T = 3.16 x 10^9 K (kT = 0.273MeV) we get,

$$\frac{16}{45}\pi b^3\alpha\tilde{A}\left(\frac{\hbar}{M_p c}\right)^2\left(\frac{1}{M_p c^2\sqrt{BM_p}}\right)\cdot\sqrt{kT} = 0.9\ s^{-1}(\text{mole/cm}^3)^{-1} \qquad (6.25)$$

recalling that $\tilde{A} = Ae^{b/\sqrt{E_0}}$, where E_0 is the Coulomb barrier height. Assuming that a temperature of 3.16 x 10^9 K (kT = 0.273MeV) is sufficiently high to make the Coulomb barrier irrelevant, we re-normalise our results accordingly. Thus, our estimate of the reaction rate at any temperature T is,

$$R[T] = 0.9\frac{\sqrt{kT}e^{-f_{min}}}{\sqrt{kT}e^{-f_{min}}\Big|_{T=3.16x10^9}} = 17\sqrt{kT(MeV)}e^{-f_{min}}\ \ s^{-1}(\text{mole/cm}^3)^{-1} \qquad (6.26)$$

For energies comparable to the Coulomb barrier height, we would expect this result to be reasonably close to that obtained from the monochromatic reaction rate based on the average energy (E = 3kT/2) and the average relative speed (~1.6√2kT/M, from Equs.1, 7). The check is carried out against the numerical calculation of the matrix element and hence cross-section, as presented in detail in Tutorial Appendix A4. Thus, at T = 3.16 x 10^9 K, i.e. for average E = 0.409MeV, numerical solution of the Schrodinger equation gives a cross section of 1.06 x 10^{-8} barns [NB: 1 barn = 10^{-28} m^2]. The mean relative velocity is 1.15 x 10^7 m/s. The reaction rate at this well defined single energy is thus,

1.06 x 10^{-8} x 10^{-28} m^2 x 1.15 x 10^7 m/s x 6.03 x 10^{29} /m^3 = 7.37 s^{-1}(mole/cm^3)$^{-1}$ (6.27)

which compares with the analytic value of 0.9 from (6.25, 6.26). Whilst this is not too bad, at lower temperatures our analytic approximation for the cross-section, (6.5), is poor. This can be seen by comparing the results of using (6.5) against the numerical results from Appendix A4:-

pp Capture Cross-Section (Barns) (B = 2MeV assumed)

E (MeV)	Numerical (App.A4)	Equ.(6.5)	Equ.(6.5) / Numerical	B/E * Equ.(6.5) / Numerical
1	3 x 10^{-8}	10^{-8}	0.3	0.7
0.1	10^{-9}	10^{-10}	0.1	2
0.01	10^{-12}	3 x 10^{-14}	3 x 10^{-2}	6
0.001	7 x 10^{-19}	2 x 10^{-22}	3 x 10^{-4}	0.6
0.0005	10^{-22}	8 x 10^{-27}	8 x 10^{-5}	0.3

The last column shows that amending our analytic estimate of the cross-section, (6.5), by a factor of B/E results in good agreement with the numerical results across a wide range of energies. Thus our improved analytic estimate is,

$$\sigma_{cap}^{E} = \frac{64\pi\alpha}{15}\cdot(\hbar c)^2\cdot\frac{B^{3/2}E^{1/2}}{(M_p c^2)^3(E+B)}e^{-b/\sqrt{E}} \qquad (6.5b)$$

The derivation of the reaction rate for a thermal distribution of energies goes through as before, except that the final result must be multiplied by a correction factor of $B/\varepsilon_0 kT$. This results in (6.24) being replaced by,

$$R[T] \approx \frac{16 \cdot 2^{2/3}}{45} \pi b^{7/3} \alpha \tilde{A} \left(\frac{\hbar}{M_p c} \right)^2 \left(\frac{c}{M_p c^2} \right) \cdot \sqrt{\frac{B}{M_p c^2}} \cdot \frac{\exp\{-f_{min}\}}{(kT)^{1/6}} \qquad (6.24b)$$

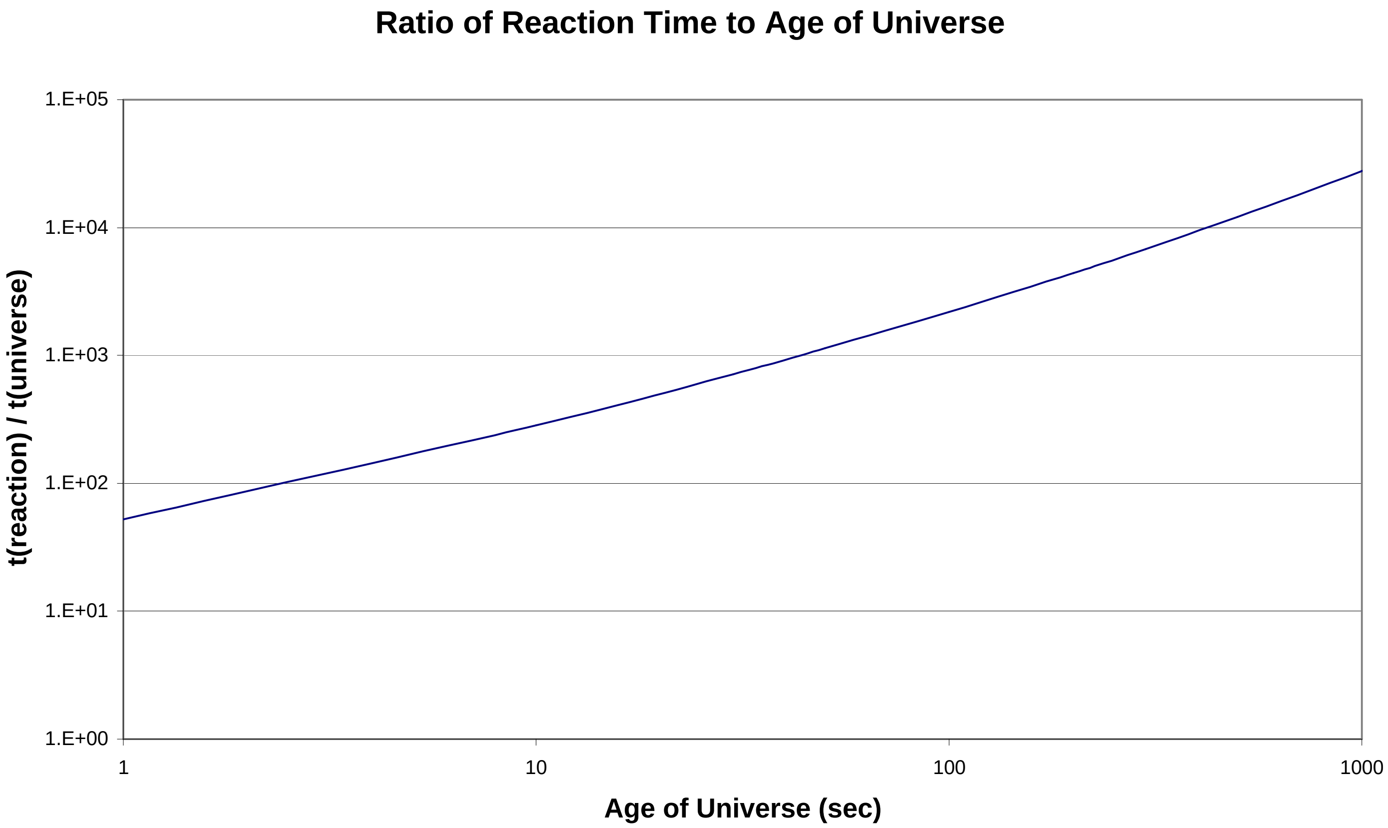
Thus, in place of (6.26), which results from (6.5), we have as a result of (6.5b),

$$R[T] = 6.7 \frac{e^{-f_{min}}}{[kT(MeV)]^{1/6}} \; s^{-1}(mole/cm^3)^{-1} \qquad (6.26b)$$

Thus, since the coefficients in the two expressions are of the same order, the two estimates of the reaction rate are not too different at high temperatures when kT is of order 1 MeV. However, at low temperatures they differ significantly. For example, at 1.5 x 10^6 K (kT = 0.000129 MeV), (6.26) gives R = 3 x 10^{-14} $s^{-1}(mole/cm^3)^{-1}$, whereas (2.26b) gives R = 4 x 10^{-12} $s^{-1}(mole/cm^3)^{-1}$, some two orders of magnitude different. The more rapid rate, from (2.26b), is the valid approximation since it agrees with the numerical calculation of the cross-section quite well.

We thus find the reaction rate during the first hour or so after the Big bang as follows,

t (sec)	T (K)	kT (MeV)	fmin	X	Reaction Rate $s^{-1}(mole/cm^3)$	proton density $/m^3$	Reaction rate /s	Reaction Time, s	Reaction time / t
1	1.E+10	9.E-01	2.E+00	2.E-01	1.E+00	8.E+27	2.E-02	5.E+01	5.E+01
10	3.E+09	3.E-01	2.E+00	1.E-01	8.E-01	3.E+26	4.E-04	3.E+03	3.E+02
30	2.E+09	2.E-01	3.E+00	9.E-02	6.E-01	5.E+25	5.E-05	2.E+04	7.E+02
50	1.E+09	1.E-01	3.E+00	7.E-02	5.E-01	2.E+25	2.E-05	6.E+04	1.E+03
100	1.E+09	9.E-02	3.E+00	5.E-02	3.E-01	8.E+24	5.E-06	2.E+05	2.E+03
200	7.E+08	6.E-02	4.E+00	4.E-02	2.E-01	3.E+24	1.E-06	9.E+05	4.E+03
300	6.E+08	5.E-02	4.E+00	3.E-02	2.E-01	2.E+24	5.E-07	2.E+06	7.E+03
500	4.E+08	4.E-02	4.E+00	2.E-02	1.E-01	7.E+23	2.E-07	6.E+06	1.E+04
1000	3.E+08	3.E-02	5.E+00	1.E-02	9.E-02	3.E+23	4.E-08	3.E+07	3.E+04
2000	2.E+08	2.E-02	6.E+00	7.E-03	5.E-02	9.E+22	7.E-09	1.E+08	7.E+04
3000	2.E+08	2.E-02	6.E+00	5.E-03	3.E-02	5.E+22	3.E-09	4.E+08	1.E+05
5000	1.E+08	1.E-02	6.E+00	3.E-03	2.E-02	2.E+22	8.E-10	1.E+09	3.E+05



Recall that a reaction becomes frozen out if its reaction time exceeds the cosmic expansion rate, i.e. if the final column of the Table (plotted in the graph) exceeds ~2. Hence the Table shows that the pp capture reaction to form diprotons is frozen out for all times after ~1 second (and actually rather earlier than that). The diproton becomes stable only for t > 35 seconds (for $g/g_{actual} \leq 1.4$). It is clear, therefore, that the diproton formation reaction is frozen out before the diproton is even stable – and hence no significant numbers of diprotons are ever formed.

7. Are Stars Possible in A Universe With Stable Diprotons?

It is not an easy matter to demonstrate conclusively that stars would be possible in any alternative universe. To do so would require a complete stellar model to be built. It would have to show that all physical constraints and equations were observed at all points in the star. This is a major exercise even in this universe. Moreover, even if a complete steady-state stellar model were developed, to be rigorous there would also need to be a demonstration that such an object could and would actually form. Given that our knowledge of star formation, even in this universe, is seriously incomplete, we are forced to conclude that a rigorous demonstration that stable stars would occur in an alternative universe is not possible at the present time.

On the other hand, it might well be possible to demonstrate conclusively that stars are not possible in some alternative universe. Of course, it is not enough to observe that stars would not be possible with the same density and temperature as stars in this universe. Such conditions would be explosive with the far faster reaction rates provided by the electro-strong proton-proton capture to form diprotons. But what about stars with lower temperatures, and perhaps lower densities, to reduce the rate of these reactions to manageable levels? Whilst we will not be able to show conclusively that such stars are possible, fortunately the burden of proof for adherents of the anthropic principle lies in the reverse. Popular texts give the impression that stars would not be possible, but without any clear demonstration. For our purposes it suffices as a refutation of their position merely to point out that the most obvious criteria for stellar stability can be met despite the initial diproton reaction.

Three stellar structure equations will be of use to us. These are derived in the next sub-section.

7.1 Constraints on Stellar Structure

Gravitationally Bound - The Virial Theorem

The first equation is a relationship between mass, density and temperature. It is a consequence of the star being gravitationally bound, and hence obeying the Virial Theorem. It states that the total kinetic energy of the particles comprising the star equals $-{}^1/_2$ of the total gravitational potential energy. For simplicity, assuming an all-hydrogen star, if the number of protons is N, the total kinetic energy of the protons plus the equal number of electrons is $2N \times 3/2kT_{av} = 3NkT_{av}$. This can be equated to $-{}^1/_2$ times the PE, i.e.,

$$3NkT_{av} = \frac{1}{2}\eta\frac{GM^2}{R} \quad (7.1)$$

where η is a number of order unity, depending upon the mass distribution. [For a uniform density sphere it would be 3/5, and hence will be larger than 3/5 for a collapsed mass of gas]. Now $M = NM_p$ so that we have,

$$kT_{av} \approx \frac{1}{10}\frac{GMM_p}{R} \quad (7.2)$$

Using $4\pi R^3\rho_{av}/3 = M$ to eliminate R in (7.2) we get,

$$kT_{av} \approx 0.16GM_pM^{2/3}\rho_{av}^{1/3} \quad (7.3)$$

It is the average temperature and density which appear in (7.3) because of the origin of the expression in the Virial Theorem, which concerns total energies. However, we allow ourselves a little license and shall interpret (7.3) in terms of the central quantities, i.e.,

$$kT_c \approx 0.24GM_pM^{2/3}\rho_c^{1/3} \quad (7.4)$$

where we have also adjusted the numerical coefficient to agree with solar central values.

Dynamic Stability

The second of our formulae concerns stability when radiation pressure becomes significant. This has been discussed in the Tutorial Chapter 15. Instability occurs if the radiation pressure is much larger than the gas pressure. Tutorial Chapter 15 shows that an assumed stability limit of $P_{rad} < 2P_{gas}$ becomes a lower limit on the density,

$$\rho > \rho_{min} = 0.11 M_p \left(\frac{kT}{\hbar c}\right)^3 \tag{7.5}$$

Substituting (7.5), at the centre, into (7.4), allows the temperature to be cancelled and thence reproduces the upper bound to stellar masses,

$$M < M_{max} = \left(\frac{8.7}{\alpha_G}\right)^{3/2} M_p \quad \text{where,} \quad \alpha_G = \frac{GM_p^2}{\hbar c} = 5.88 \text{x} 10^{-39} \tag{7.6}$$

Hence, the upper bound stellar mass is $\sim 47 M_{sun}$.

Maximum Radiative Heat Transport Rate

The maximum rate at which heat can be transported through a medium of a given opacity and density defines a maximum power density that can be supported in steady state by purely radiative heat transfer. From the Tutorial Chapter 16, the radiative heat transport equation is,

$$\frac{dT}{dm} = -\frac{3}{256\pi^2\sigma_{SB}} \cdot \frac{\kappa L}{r^4 T^3} \tag{7.7}$$

The radiation pressure is, $$P_{rad} = \frac{4\sigma_{SB}}{3c} T^4 \tag{7.8}$$

Hence, $$\frac{dP_{rad}}{dm} = -\frac{\kappa L}{16\pi^2 c r^4} \tag{7.9}$$

By hydrostatic equilibrium: $$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \tag{7.10}$$

Which give, $$\frac{dP_{rad}}{dP} = \frac{\kappa L}{4\pi c G m} \tag{7.11}$$

Now the total pressure is the sum of the gas pressure and the radiation pressure,

$$P = P_{gas} + P_{rad} \tag{7.12}$$

Since the gas pressure and radiation pressure are both positive, and since both reduce monotonically with increasing r, it follows that (7.11) is < 1. Hence we get,

$$L < 4\pi c G m / \kappa \tag{7.13}$$

This is true at all points in the star. In the particular case of a point near the centre, the luminosity is,

$$L_{\text{small r}} = \frac{\varepsilon_{v,c}}{\rho_c} m_{\text{small}} \tag{7.14}$$

Hence,

$$\varepsilon_{v,c} < 4\pi cG\rho_c / \kappa \tag{7.15}$$

In passing we note that the general expression, (7.13), can be written in terms of the average power density per unit mass,

$$\langle \varepsilon_m \rangle < 4\pi cG / \kappa \tag{7.16}$$

where the average is with respect to mass, i.e.,

$$\langle \varepsilon_m \rangle = \frac{1}{m}\int_0^m \varepsilon_m dm \tag{7.17}$$

However, for our present purposes it is the simpler Equ.(7.15) which we shall consider.

7.2 Example Star Burning Via Diprotons

Equ.(7.4) shows how to find the mass of a star if the central temperature and density are known. Equ.(7.5) defines the lowest density which results in a stable star given the temperature. We will interpret Equ.(7.5) as relating to central values. Finally, Equ.(7.15) defines the maximum power density that is possible for a given density and opacity. Since the power density is proportional to the proton density squared (because the first reaction is two protons reacting together), for a given temperature Equ.(7.15) defines a maximum density for which radiation suffices to carry away all the heat produced near the centre of the star.

Now, assuming that stars are indeed possible in this alternative universe, there will be a unique central temperature and density corresponding to any given total mass within the possible stellar range[1]. Determining what the central temperature and density are for a given mass requires a complete stellar solution, respecting hydrostatic equilibrium and heat balance at all points. This is a larger task than we intend to address. Instead, we shall assume a central temperature. We will then consider the lower and upper bound densities permitted by (7.5) and (7.15) respectively.

We shall consider a central temperature of $\mathbf{10^6}$ **K** (kT= 0.000086MeV).

7.2.1 Lower Bound Density

The lower bound density from (7.5) is 0.015 kg/m^3 (ρ_p^N = 7 x 10^{24} /m^3 = 1.1 x 10^{-5} mole/cm^3).

[1] That is, assuming a Main Sequence star in the alternative universe, i.e. one burning hydrogen in steady state.

Reaction [1]: $p + p \rightarrow {}^2_2He + \gamma$ **:-**

The reaction rate is given by (6.26b). We have b = 0.7024√MeV and f_{min} = 33.82 hence R = 6.6 x 10^{-14} s^{-1} $(mole/cm^3)^{-1}$. Hence the reaction rate per proton is 7.5 x 10^{-19} s^{-1}. The reaction time is thus ~4 x 10^{10} years. Hence, assuming this is the rate determining reaction, the lifetime of the star is comparable with that of solar mass stars in this universe, and as such compatible with the time-scales for the evolution of biological life. The example temperature of 10^6 K was contrived to achieve this result.

We shall assume that the weak decay of the diproton to deuterium is sufficiently fast so as not to influence the reaction kinetics.

We now need to determine if the nuclear reactions continue to produce helium-3 and helium-4. This will influence the total heat production. We also need to find out if the reactions producing these subsequent nuclei are faster or slower than the initial diproton reaction.

Reaction [2]: $p + D \rightarrow {}^3_2He + \gamma$ **:-**

We do not know how the increase in the strength of the nuclear force will affect nuclear reaction rates. However, reactions producing a gamma photon as a product are clearly mediated by the electromagnetic interaction. We can reasonably assume their rate is unaffected to first order. This reaction has been studied in detail from first principles in Appendix A3 and the results compared with the data of Hoffman et al. The factor $X = e^{-f_{min}} / (kT)^{1/6}$ is used as the basis of extrapolation to 10^6 K:-

Reaction Rates for [2]: $s^{-1}(mole/cm^3)^{-1}$ [b = 0.811 √MeV]

T (10^6 K)	My Calc (App.A3)	Hoffman et al	X	Scaled using X from 50 x 10^6 K
1.0			3.2 x 10^{-16}	3.4 x 10^{-12}
1.5			3.32x10^{-14}	3.6x10^{-10}
10	0.001		1.03x10^{-7}	0.0011
14	0.0053		0.817x10^{-6}	0.0088
30	0.127	0.21	1.7x10^{-5}	0.184
50	0.71	1.10	1.03x10^{-4}	(1.10)

Subsequent Reactions:

There are now a number of competing possible reactions, as listed below together with their rates at 10^6 K taken directly from Hoffman et al (except for [g] where an extrapolation to lower temperatures was required, for which the factor $X = e^{-f_{min}} / (kT)^{1/6}$ has again been used):-

		Rate at 10^6 K $s^{-1}(mole/cm^3)^{-1}$
[1a]	$p + p \rightarrow {}^2_2He + \gamma$	6.6 x 10^{-14}
[1b]	${}^2_2He \rightarrow {}^2_1D + e^+ + \nu_e$	*Assumed fast*
[2]	$p + D \rightarrow {}^3_2He + \gamma$	3.4 x 10^{-12}
[3]	${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1p$	1.46 x 10^{-41}
[a]	${}^2_1D + {}^2_1D \rightarrow {}^4_2He + \gamma$	7.85 x 10^{-16}
[b]	${}^2_1D + {}^2_1D \rightarrow {}^3_1H + {}^1_1p$	6.72 x 10^{-9}
[c]	${}^2_1D + {}^2_1D \rightarrow {}^3_2He + {}^1_0n$	6.33 x 10^{-9}
[d]	${}^3_1H + {}^2_1D \rightarrow {}^4_2He + {}^1_0n$	1.88 x 10^{-7}
[g]	${}^3_1H + {}^1_1p \rightarrow {}^4_2He + \gamma$	*7.5 x 10^{-12}**
[i]	${}^3_2He + {}^2_1D \rightarrow {}^4_2He + {}^1_1p$	3.84 x 10^{-19}
[j]	${}^3_2He + {}^3_1H \rightarrow {}^4_2He + {}^2_1D$	1.42 x 10^{-22}
[k]	${}^3_2He + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + {}^1_1p$	2.0 x 10^{-22}

**Extrapolated from Hoffman et al's data at 30 x 10^6 K.*

Since [c] and [d] and [k] create neutrons, there could also be other reactions involving neutrons as reactants. There will also be reactions involving higher mass nuclei, analogous to the ppII and ppIII sequences. We shall ignore these for simplicity.

Reactions [i], [j], [k] are clearly far slower than the dominant reactions at this low temperature, and this can be traced to the Coulomb barrier which is higher for these reactions since they involve a doubly charged reactant (helium-3). Similarly, reaction [3] is by far the slowest, since this involves double charges on both reactants. Scaling from reaction rates at a higher temperature (3 x 10^8 K) on the basis of the X-factor confirms the low temperature reactions rates given above, to within an order of magnitude. Reaction [a] is also too slow to contribute. In summary, the reactions shaded yellow above are negligible, whereas reaction [1b], shaded red, is so fast it plays no part in the kinetics. We therefore need consider reactions [1a], [2], [b], [c], [d] and [g] only.

We shall use a notation in which [r] denotes the rate of reaction 'r', and p, D, t represent the number density of protons, deuterons and tritium. In the steady state, equating the rate at which deuterons are produced to the rate at which they are consumed gives,

$$[1a]p^2 = [2]pD + [b]D^2 + [c]D^2 + [d]Dt \quad (7.18a)$$

Similarly, equating the rate of tritium production with the rate of its consumption gives,

$$[b]D^2 = [d]Dt + [g]pt \quad (7.18b)$$

A little trial and error shows that the solution to these simultaneous equations is,

$$\frac{D}{p} = 0.00175 \qquad \text{and} \qquad \frac{t}{D} = 0.035 \tag{7.18c}$$

noting that these solutions apply for any absolute density (i.e. any p).

Assuming our above estimate of the life of the star, ~40 Byrs, based solely on reaction [1a] is about right, the total rate of helium-3 and helium-4 production (via reactions 2+c and d+g respectively) suggest that the final number density of helium-3 is about 36% of the initial proton number density, and similarly that of helium-4 is about 30% of the initial proton number density.

Actually these are not possible figures since all the protons will be used up if either the helium-3 density reaches 33% or the helium-4 density reaches 25%, or if the two reach (say) 16% and 13% respectively. Hence, the end of life of the star is only about half our initial crude estimate, i.e. about 20 Byrs. This is even closer to that of a solar mass star in this universe, so all's well and good. Strictly, of course, this is just the length of the hydrogen burning phase.

Moreover, the above calculations suggest that: (a)All the protons are converted to helium-3 or helium-4 by the end of the hydrogen burning phase; (b)There is a roughly equal amount of helium-3 and helium-4 in the star's core after this time. This suggests that subsequent gravitational collapse would result first in a helium-3 burning phase (via reaction [3]), and only some time later would the usual helium-4 burning phase occur.

Power Density (assuming the lower bound density at 10^6 K)

The preceding considerations have shown that the reactions proceed to a roughly equal mix of helium-3 and helium-4, and that the rate is determined by the first reaction, at least to within a factor of two or so. The heat energy that is released for every helium-4 nucleus formed is its binding energy, less twice the mass difference between the neutron and proton. [The neutrinos carry away a little energy, but this is negligible]. What is the binding energy of helium-4 in our modified universe? We estimate this using Equ.(13), together with the actual binding energy of helium-4 in this universe, i.e. 28.3 MeV.. We thus estimate heat production to be 151, 252, 378 MeV per helium-4 nucleus for g_s increases of x1.2, x1.3, x1.4 respectively. As an upper bound on the power, we assume complete conversion to helium-4. Hence, per pp capture reaction the heat released is not more than 75, 126, 189 MeV.

Given the reaction rate and proton density assumed above, we thus find a power density of around,

$$\begin{aligned}\varepsilon_V &= [86.6\text{x}10^{-14}\ \text{s}^{-1}\ \text{x}\ (6.9\text{x}10^{24})^2\ \text{m}^{-3}/\ 6\text{x}10^{29}]\ \text{x}\ (75 - 189\ \text{MeV}) \\ &= 0.00006 \text{ to } 0.00016\ \text{W/m}^3\end{aligned}$$

Maximum Stable Power Density

The maximum power at the centre of the star if the heat is to be transported away by radiation alone is given by (7.15), i.e.,

$$\varepsilon_{v,c} < 4\pi cG\rho_c / \kappa = 0.11\ \text{W/m}^3$$

Hence our estimated power density, assuming the minimum density, is well within that which can be transported away by radiation.

In the above estimate of the maximum stable power density we have assumed that the opacity is given purely by Thompson scattering, i.e. $\kappa = 0.034\ m^2/kg$. This is the lower bound opacity at high temperatures. It applies only if the medium is fully ionised. Even small proportions of nuclei with captured electrons will markedly increase the opacity (giving rise to Kramer's opacity instead). Thus, at around 10^5 K the opacity can be six orders of magnitude larger, depending upon density. Indeed, at our chosen temperature of 10^6 K the opacity would be three orders of magnitude larger if the density were around 1000 kg/m^3 instead of ~0.015 kg/m^3. As it is, our chosen temperature together with our chosen low density is just consistent with the lower bound (Thompson) opacity.

Thus our chosen conditions respect both of the stellar structure constraints, (7.5) and (7.15), discussed above.

Size, Mass, Luminosity and Surface Temperature of Our Hypothetical Star (with $T_c = 10^6$ K and $\rho_c = 0.015\ kg/m^3$)

The mass of the star follows from the central temperature and density using Equ.(7.4) and is 9.5 x 10^{31} kg = 47.5 solar masses. This is the upper bound stellar mass which is stable against disruption by radiation pressure. (This was the basis of the minimum density assumption, so this is just a consistency check).

Without a detailed structure model we cannot deduce the radius, luminosity or surface temperature of the star. For these quantities we make some crude estimates.

Firstly, the luminosity will be estimated based on the assumption that one-quarter of the star's hydrogen is involved in nuclear reactions, and that these all occur at central rates. The former assumption derives from solar models, whereas the latter assumption will probably result in an over-estimate of the luminosity. The power per proton in the centre is (0.00006-0.00016) Wm^{-3}/ 6.9 x 10^{24} m^{-3}. Multiplying by one-quarter of M/M_p gives a luminosity of 1.3 – 3.3 x 10^{29} W, or about 320 to 780 times solar luminosity. [NB: It is well within the Eddington luminosity limit of 7 x 10^{32} W for a star of this mass].

Secondly, to estimate the radius we again proceed by analogy with solar models. Specifically, the Clayton model has $M = \sqrt{6}\left(4\pi a^3 \rho_c / 3\right)$ where a reasonable fit to the Sun is a ~ R/5.4. Hence we find,

$$R \approx 2.485\left(\frac{M}{\rho_c}\right)^{1/3} \tag{7.19}$$

Hence we estimate R ~ 4.5 x 10^{11} m, or about 650 times the solar radius. (This is comparable with the size of a red giant in this universe).

Equating the total surface heat emission with the luminosity then implies a surface temperature of between 1,400 K and 1,740K. Thus, the great size of the star leads to it being rather 'cold' in terms of the frequency of the emitted radiation, despite its great luminosity. The peak in the star's spectrum lies between 29,000 and 36,000 Angstroms, and hence is well outside the visible range (roughly 4,200-7,000 Angstroms). However, the peak in the Sun's spectrum is also just beyond the visible band (~8,600 Angstroms). The most common photons from the star have energy quanta around ~0.4eV, compared with ~1.4eV from the Sun. Such energies are not obviously incompatible with the anthropic requirement to induce the molecular reactions associated with biological life. But what exactly is that requirement?

7.2.2 Upper Bound Density

Recall that, for our arbitrarily chosen central temperature of 10^6 K, we initially chose to consider the minimum density compatible with dynamic stability. We now attempt the opposite bound – the largest density consistent with the upper limit on power density given by Equ.(7.15).

Since power density depends upon density squared, whereas the limiting power density is proportional to the density, to increase our power density to a limiting value apparently requires an increase in density by a factor 0.11 W/m^3 / 0.00016 W/m^3 = 687, i.e. to 10.5 W/m^3. However, the greater density will cause the opacity to increase and this will reduce the upper limit on power density. Establishing the maximum density is therefore an iterative matter. We find that a factor of 170, to give a density of **2.6 kg/m^3**, yields an opacity of 0.145 m^2/kg, and this produces a limiting condition, i.e. a power density equal to the maximum power density, both being 4.5 W/m^3 (NB: we are now using only the upper estimate for the nuclear heating rate).

Thus, using a proton density of 1.5 x 10^{27} m^{-3} we find that the controlling pp capture reaction time, i.e. the star's lifetime, is ~200 Myrs. The star's mass is ~7 x 10^{30} kg, or around 3.6 solar masses. It's radius is ~3.5 x 10^{10} m, or around 50 solar radii. It's luminosity is 3 x 10^{30} W, or about 7,500 times that of the Sun. Finally, its surface temperature is about 7,660K. Thus, the two extremes of density result in surface temperatures which fall either side of the solar surface temperature (5900K).

7.3 Summary of Section 7

In reality there will be a unique central density which corresponds to an assumed central temperature. Both will be uniquely related to the star's mass. Unfortunately it is not simple to determine what this unique central density is. This is because it arises from the requirement to satisfy hydrostatic equilibrium and heat balance everywhere within the star. Hence a complete stellar model is required to determine the correspondence between central temperature and density. However, we have shown that there is a lower and an upper bound to the central density for a given central temperature. By considering these limiting cases, we conclude that a stable star with a central temperature of 10^6 K appears to be credible in a universe with stable diprotons. The properties of such a star would be:-

- Lifetime: 200 Myrs to 20 Byrs;
- Luminosity: 300 to 7,500 times L_{sun};
- Mass: 3.5 to 47 M_{sun};
- Radius: 50 to 650 times R_{sun};
- $T_{Surface}$: 1,400 K to 7,600 K;

Hence, stars in such a universe would be larger, more massive and more luminous, than stars with a similar lifetime in this universe. Whether they would be cooler or hotter cannot be determined without a complete stellar model.

Most importantly, it appears not to be difficult to achieve stars with lifetimes in the region of billions of years, as required for biological evolution. The luminosity and surface temperature may well be of the correct order to support biological life based on conventional molecular chemistry.