Chapter 8B – The Hoyle Coincidence Part 2: The Quantity of Carbon and Oxygen Produced by Stars

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TRACK 1

1. Introduction

In Chapter 21 of the Cosmology Tutorial we showed, in a highly simplified manner, how comparable quantities of carbon and oxygen could result from helium burning in red giant stars. We showed how the key nuclear reaction rates could be estimated from the energy levels of certain nuclear resonances. We also demonstrated, albeit very crudely, that reducing the rate of the triple alpha reaction by a factor of 100 would result in carbon levels of only a fraction of a percent of their actual values. Similarly, increasing the rate of the triple alpha reaction by a factor of 100 would result in oxygen levels reducing to a few percent of their actual values. Alternatively, changing the rate of the subsequent reaction, the capture of a further alpha particle to form oxygen, would have a similar effect. However, all these conclusions are based on a massively oversimplified model of the conditions within such a star.

In reality, the conditions prevailing within red giant stars are extremely complex. This is evident from the description of their evolution in Chapter 17 of the Cosmology Tutorial. Despite our attempts in Chapter 21, it is not really possible to give an adequate account of carbon and oxygen formation in red giant stars unless a modern stellar evolution code is deployed. Even if it were, it would not be sufficient to address the question of interest. This is, "how much carbon and oxygen is released into the ISM?" A complete account must also model the mechanisms by which the nuclear reaction products are ejected into the ISM. There are two such mechanisms: the stellar wind and supernovae. Moreover, the stellar wind provides a mechanism for element dispersion only by virtue of the various "dredge-ups" bringing the reaction products to the outer parts of the stars. Consequently, it is also necessary to model these dredge-ups, which requires calculation of the convective boundary and how it undergoes transient movements. Thus, the more comlex phenomena of stellar evolution become essential in predicting the abundance of elements in the ISM. Regrettably, such predictions are beyond the scope of simple 'hand' estimates based on a few simple equations. They require sophisticated numerical stellar codes.

This Chapter addresses the following:-

- Firstly we describe the Hoyle Coincidence in qualitative terms.
- Secondly, we examine evidence from the literature, based on detailed stellar models, regarding the sensitivity of carbon and oxygen abundance to the rate of the triple alpha reaction. We shall see that the carbon and oxygen abundances are not as sensitive to the reaction rates as the simple arguments of Chapter 21 suggested.
- Nevertheless, the very great sensitivity of the triple reaction rate to the strength of the nuclear force, as discussed in Chapter 8A, implies that the amount of carbon and oxygen produced would almost certainly change by orders of magnitude for changes in the nuclear force of a fraction of a percent.
- However, it is pointed out that the Hoyle coincidence is only a one-sided fine-tuning. It the strength of the nuclear force were changed sufficiently to make Be^8

stable, this changes the game completely. A qualitatively new phase in stellar evolution would occur, in which both carbon and oxygen could be made by processes which, in this universe, do not occur – such as $Be^8 + Be^8$ reactions.

- We will conclude that the Hoyle coincidence remains a rather curious bit of fine-tuning, but is considerably weakened by virtue of being single-sided only.

2. The Hoyle Coincidence Restated

The Hoyle coincidence is claimed to consist of three sequential coincidences. The first is that Beryllium-8 can be formed resonantly from two alpha particles, i.e. its energy is just slightly greater than the two alpha threshold (by 91.9 keV). Because Be^8 is unstable, in order for there to be a significant equilibrium concentration of Be^8 it is necessary that its formation reaction is very fast. The exponential dependence of resonant reaction rates on the resonance energy (above threshold) means that if the energy of Be^8 were (say) twice as great, then the reaction rate would be fatally slower.

In order to convert a very low Be^8 concentration into a significant carbon concentration, the capture of a further alpha particle also has to be resonant. The second coincidence is that there is indeed a resonance of C^{12} at an appropriate energy to make this reaction resonant. The existence of this resonance was predicted by Hoyle for this reason. It was actively sought and found following his prediction in 1953. This 'missing link' permitted Hoyle to finally solve the stellar nucleosynthesis problem in his classic 1954 paper, Reference [15].

Finally, to avoid all the carbon being immediately cooked into oxygen by a further alpha particle capture, it is necessary that this reaction is *not* resonant. But the rate of alpha particle capture by carbon must result in a balance of carbon and oxygen production. The third coincidence is that the $\alpha + C^{12} \to O^{16}$ reaction misses a resonance by just 45 keV, fulfilling these conditions.

The coincidences are nicely illustrated by the energy levels plots given in Chapter 8A.

The importance of these resonance energies is that the reaction rates are extremely sensitive to them. A change in the $\alpha + {}^{8}_{4}Be \to {}^{12}_{6}C + \gamma$ reaction rate can be translated into a change in the C^{12} 0_2^+ resonance energy, and vice-versa. From Chapter 21, Equation (21.4.3.13), we see that the rate of the reaction $\alpha + {}^{8}_{4}Be \to {}^{12}_{6}C + \gamma$ depends upon two parameters of the resonance: its energy and its partial width for emitting a gamma. The dependence on the latter is merely linear, however, and since the partial width changes only by a few percent for the variations we consider, it is quite irrelevant to the level of accuracy we are concerned about. The resonance energy dependence of the reaction rate, on the other hand, is exponential. Hence, the change in reaction rate is simply,

$$\text{Factor by which reaction rate changes} = \exp\{-\Delta E_R / kT\} \qquad (8B.1)$$

In particular, reducing the resonance energy will increase the reaction rate, and vice-versa. We can now determine what change in resonance energy corresponds to a factor of 100 change in reaction rate. It is temperature dependent. At 100 MK it is 40 keV. At 150 MK it is 60 keV.

Conversely, a change in the resonance energy of ±100 keV corresponds to a reaction rate change (at 150 MK) of exp(-100/12.94), i.e. of the order of 4×10^{-4} or 2×10^{3}. A change in the resonance energy of ±150 keV corresponds to a reaction rate change (at 150 MK) of exp(-150/12.94), i.e. of the order of $10^{+/-5}$. It is, of course, hardly surprising that a change in a key reaction rate of five orders of magnitude has rather drastic implications for the abundance of carbon and oxygen.

Can plus-or-minus five orders of magnitude (which means a range of ten orders of magnitude) count as fine-tuning? The point is that the arguments of Chapter 8A imply that changes in the resonance energy of the required order (~150keV) are almost inevitable if the strength of the nuclear force were changed even by just a fraction of a percent. So, yes, it does count as fine-tuning. What is fine-tuned is the dependence of the reaction rate on the strength of the nuclear force, not the dependence of the carbon and oxygen abundances on the reaction rate.

So, are the biological requirements for sufficient release of carbon and oxygen into the ISM prejudiced by a change in the strength of the nuclear force by a fraction of a percent - or not? We now turn to the results of stellar models.

3. Qualitative Considerations and Early Stellar Models

As we have already remarked, the literature does not support quite so sensitive a dependence of carbon/oxygen production on the resonance energy as our simplistic model in Chapter 21 of the Tutorial suggests. It is worth briefly examining the reasons for this:-

- Most obviously, our crude model did not include all the relevant reactions. We did not even account for oxygen consumption, e.g. by further alpha capture.
- We assumed reaction at a single temperature, and involving a fixed body of material. This may be crudely representative of the initial helium burning in the core, but is otherwise wrong in two ways…
- Firstly, after the core helium is exhausted, helium burning continues in a shell around the core. This shell burning prevails whilst the star climbs the asymptotic giant branch (AGB). Because the shell is thin, and hence contains only a limited amount of helium, the helium can become exhausted by the time substantial amounts of carbon have formed. This gives less opportunity for subsequent oxygen formation in the shell.
- Secondly, we are not really dealing with a fixed body of material undergoing reactions to completion. Periodically, convective mechanisms will convey material which is still reacting to the outer envelope of the star. Here temperatures are too low to support nuclear fusion and the reactions cease. In this way, carbon or oxygen can be saved from destruction and stored in the envelope for subsequent ejection into the interstellar medium.
- A phenomenon called "helium flash" provides a mechanism for such convective mixing of the fusion products, in low and intermediate mass stars. A star climbing the AGB will have concentric helium and hydrogen burning shells, the latter feeding helium into the former. Periodically, the helium burning becomes 'run away', i.e. neither the rate of heat transfer cannot balance the rate of heat production. Consequently a thermal instability arises between the shells. This results in the envelope's convective boundary penetrating into the region of core

fusion products, and dredging them up into the envelope. This is the so-called "third dredge-up".

- If the rate of the triple alpha reaction is reduced, but not by too much (say, 60-100 keV increase in the C12 0_2^+ resonance energy) helium flashes will still occur. However, the reduced rate of the triple alpha reaction is partly compensated by the fact that the helium flash will now occur only at higher pressure, density and temperature. In addition, a more massive convective shell is produced. Hence the mechanism of transferring carbon and oxygen to the envelope can be more efficient and hence partly compensate for the reduced reaction rate.

In passing we note that the relative contributions of low ($<1.5M_\odot$), intermediate and high mass ($>10M_\odot$) stars to the overall cosmic abundance of carbon appears to be contentious, see for example Prantzos, Reference [4]. Gustafsson et al (1998, Reference [5]) have suggested that cosmic carbon is dominated by the production in heavy stars. On the other hand, Chiappini and others, References [6] and [7], favour large amounts of carbon being produced by low or intermediate mass stars, in the case of sufficiently high metallicity. The proportion of carbon arising from low, intermediate and high mass stars depends sensitively on the metallicity of the star (see for example Reference [4], Figure 6). For low metallicity stars, only massive stars release large amounts of carbon into the interstellar medium. The earliest stars (so-called population III) are particular cases. Since little or no metal elements had had time to form when such stars were born, they would have had extremely low metallicities. They also tend to have anomalously high levels of carbon, see References [8, 9, 10]. The likelihood is that most of the earliest, population III, stars were extremely massive (possible not merely tens of $M_\odot$ but hundreds of $M_\odot$). However, Prantzos, Reference [8], argues that we can expect some 'normal' low mass stars in population III also. The import of these observations is that it is necessary to consider low, intermediate *and* high mass stars in evaluating the sensitivity of carbon/oxygen production to the triple alpha reaction rate. I come to the opposite conclusion below. Which is right?

An early investigation by Livio et al, Reference [11], considered a massive ($20M_\odot$) star and an intermediate mass ($5M_\odot$) star. For the massive star, increases in the energy of the C^{12} 0_2^+ resonance of 490 keV, 277 keV and 60 keV were considered. The first two of these resulted in essentially no carbon being produced. This is not surprising since an energy change of 277keV would reduce the triple alpha reaction rate by nine orders of magnitude at 150 MK (or by seven orders of magnitude at 200 MK). The 60 keV increase resulted in 5×10^{-4} $M_\odot$ of new carbon being formed. This is very low compared with the amount of carbon produced using the true reaction rate.

In the case of the $5M_\odot$ AGB star, it is necessary to use a stellar model capable of handling the thermal pulse and the other issues relevant to the outcome as regards carbon production and potential ejection into the interstellar medium. Livio et al considered two resonance energy increases: 277 keV and 60 keV. As before, the larger energy increase reduced the carbon production to virtually nothing. The smaller, 60 keV, increase is more interesting in this case. This case is claimed by Livio et al to result in little change to the carbon production, apparently due to the increased efficiency of the dredge-up. Unfortunately, Livio et al did not consider intermediate cases, i.e. increases lying between +277 keV and +60 keV, and so did not identify the resonance energy at which the carbon production would first be significantly prejudiced.

The early work of Csoto, Oberhummer and Schlattl, References [12, 13], revisited stellar models with changed rates for the triple alpha reaction. They considered a massive star ($20M_\Theta$), and intermediate mass star ($5M_\Theta$) and a low mass star ($1.3M_\Theta$). Unlike Livio et al, Csoto et al also addressed oxygen production explicitly. They considered $C^{12}\ 0_2^+$ resonance energy changes of ±105keV and ±156keV. The results of this early work by Csoto et al can be broadly summarised as follows: An increase in the $C^{12}\ 0_2^+$ resonance energy by 156 keV reduces carbon production to 3% or less of its normal level, whereas an decrease in the resonance energy by the same amount reduces oxygen production to ~1% of its normal value. These results are consistent with that of Livio et al in that 156 keV lies between the resonance energy changes of 60 keV and 277 keV which Livio et al found to produce respectively either little change or to reduce the carbon production to negligible levels.

4. Recent Stellar Models

The most recent work considering the effects of varying the $C^{12}\ 0_2^+$ resonance energy on stellar models is that of Schlattl, Heger, Oberhummer, Rauscher and Csoto, Reference [17]. They consider changes of ±100keV. However, they kept the rate of the oxygen formation reaction $C^{12} + \varepsilon \rightarrow O^{16} + \gamma$ fixed. They argued that a change of the sub-threshold O-resonance which determines the rate of this reaction by the same amount as the change in $C^{12}\ 0_2^+$ would cause the rate of $C^{12} + \varepsilon \rightarrow O^{16} + \gamma$ to change by two orders of magnitude less than that of the triple alpha reaction. This claim will be examined more closely later.

Schlattl et al [2004] claim improved stellar modelling compared with the previous work by (some of) the same authors, i.e. Refs.[12, 13]. In particular, the dredge up process in low and intermediate mass stars was modelled in greater detail, as was the explosive nucleosynthesis and ejecta behaviour during supernova in massive stars.

For $15M_☼$ and $25M_☼$ stars, the results of Schlattl et al [2004] for carbon and oxygen abundances in the ejecta of the supernova are given below in terms of a percentage of the initial stellar mass (black) and as a percentage of the normal value (blue). Figures are for $15M_☼$ ($25M_☼$) stars.

15M☼ (25M☼) stars

Element	**$C^{12}\ 0_2^+$ resonance energy**		
	-100keV	**This universe**	**+100keV**
Carbon	2.4% (10%)	0.9% (1.4%)	0.13% (0.08%)
Oxygen	0.4% (3.2%)	4.7% (12%)	2.2% (6.6%)
Carbon	280% (760%)	100%	**15% (6%)**
Oxygen	**8.5% (26%)**	100%	46% (53%)

Very roughly, an increase of 100keV in the resonance energy leads to the carbon ejected into the ISM being reduced by an order of magnitude compared with the standard case. Conversely, a decrease of 100keV in the resonance energy leads to the oxygen ejected into the ISM being reduced by between a factor of four and a factor of ten compared with the standard case. Schlattl et al [2004] regard their results as showing less sensitivity of the C and O production than their earlier models, Refs.[12, 13]. However, it could be argued, in very approximate terms, that these latest results are broadly comparable. Livio et al showed that a resonance energy increase of 60keV had little effect on C and O production, whereas Refs.[12, 13] suggested that changes of ±156keV would cause about two orders of magnitude decrease in either C or O production. Hence, the results of Schlattl et al [2004], that changes of ±100keV would cause about one order of magnitude decrease in either C or O production, could be interpreted as broadly consistent.

Schlattl et al [2004] next considered an intermediate mass star of initial mass $5M_{☼}$. In the case when the C^{12} 0_2^+ resonance energy was reduced by ~100keV, some qualitative changes in the stellar physics occur. The first is the paradoxical result that carbon abundance in the envelope, and hence in the stellar wind, are reduced. This is surprising in that a reduced resonance energy increases the triple alpha reaction rate. Consequently the carbon abundance within the reactive regions increases. The paucity of metals in the envelope is due to the fact that the third dredge-up does not occur. This is because the reduced resonance energy leads to a reduced helium ignition temperature, which leads to weaker helium flashes. It is usually the strong helium flashes which drive the transient inter-shell convection. This is the mechanism underlying the third dredge-up. Consequently, the third dredge-up does not occur because the helium flashes are too weakened.

The resulting metal-poor envelope is likely to give rise to a much reduced stellar wind. Now stars of initial mass below ~$8M_{☼}$ would usually end their life as white dwarves, rather than leading to Type II supernovae. Their contribution to the chemical make-up of the ISM would therefore depend upon the stellar wind and the abundance of species within their envelopes. For a reduced resonance energy, the low levels of reaction products in the envelope and the reduced stellar wind, would be expected to lead our $5M_{☼}$ star to contribute far less carbon and oxygen to the ISM. However, this brings us to the second possible qualitative changes in the stellar physics. In the model of Schlattl et al [2004], the reduced mass loss from the star lead to a greater core mass than normal. Schlattl et al suggested that this may result in the usual initial mass limit for Type II supernovae (~8-9 $M_{☼}$) might be reduced to below $5M_{☼}$. If true, the contribution of our $5M_{☼}$ star to the carbon abundance of the ISM would be radically greater, and, in particular, larger than the standard case.

Because of the uncertainty over whether a $5M_{☼}$ star would undergo supernova collapse, it is not possible to confidently predict whether a reduced resonance energy would lead to a smaller or a greater contribution of carbon to the ISM. This provides a good illustration of why simplistic estimates of the effects of changing a single physics parameter can mislead. Even the sense of the resulting change in element production may be contrary to naïve expectation. The complex heat and mass transport phenomena within a star may drive the change in a direction opposite from that which would be expected based on nuclear physics alone.

The case of a $5M_{☼}$ star with reduced resonance energy is simpler, though even here the effect of the changing strength of the helium flashes is significant. In this case the helium flashes are stronger, and a third dredge-up is assured. Moreover, a great deal of carbon is produced during the helium flashes, so their increased strength partly mitigates the reduced triple alpha reaction rate.

The tables below gives the carbon and oxygen abundances from Schlattl et al [2004] in terms of a percentage of the initial stellar mass (black) and as a percentage of the normal value (blue), for a 5M☼ star. For the reduced resonance energy, the first result given relates to the abundances in the envelope and wind only, whilst the second is for the whole star. The latter is relevant only if a supernova occurs. But a Type II supernova cannot occur, can it? Do Schlattl et al mean a Type Ia supernova? Do Type Ia supernovae release C and O into the ISM? I think HKS claim they do.

5M☼ stars

Element	C^{12} 0_2^+ resonance energy		
	-105keV	This universe	+94keV
Carbon	0.0003% (22%)*	0.042%	0.022%
Oxygen	0.0002% (0.3%)*	0.011%	0.054%
Carbon	0.7% (500%)*	100%	52%
Oxygen	1.5% (27%) *	100%	500%

**no supernova (with supernova)*

Hence, it is not clear whether a change in the resonance energy of ±100keV would cause a very great reduction in either the carbon or the oxygen contribution of a 5M☼ star. In particular, an increase in the resonance energy of ~100keV appears to be biologically benign.

However, we may question how much stars of mass around 5M☼ contribute to the total carbon and oxygen abundance. This depends upon the relative number of such stars compared with stars of higher mass. For example, assuming a birth function proportional to $M^{-2.35}$, stars of mass 5M☼ will out-number stars of mass 15M☼ by a factor of $3^{2.35}$ = 13.2. In this universe, every 15M☼ star contributes 0.9% x 15M☼ = 0.13M☼ of carbon, and 4.7% x 15M☼ = 0.7M☼ of oxygen. In comparison, for each such 15M☼ star, 5M☼ stars contribute 13.2 x 0.042% x 5M☼ = 0.028M☼ of carbon, and 13.2 x 0.011% x 5M☼ = 0.007M☼ of oxygen. Hence, it seems most likely that the high mass stars are the dominant contributors to the current carbon and oxygen content of the universe at large (i.e. ignoring the hearts of dead stars).

This argument becomes a great deal more convincing still when account is taken of the different lifetimes of stars of differing mass. In the lifetime of one 5M☼ star, about fifteen stars of mass 15M☼ could have yielded their quota of elements. This makes the contribution of the higher mass stars relative to the lower mass stars much greater still. Consequently we do not discuss the results of Schlattl et al [2004] for a 1.3M☼ star, since this is likely to contribute negligibly to the total abundances. Is this right?

In conclusion, the table for the 15M☼ and 25M☼ stars is probably the best guide to the effect of resonance energy changes on the abundance of carbon and oxygen in the universe at large.

5. Effect of Nuclear Force Changes on the $C^{12} + \varepsilon \rightarrow O^{16} + \gamma$ Reaction Rate

Schlattle et al [2004] kept the rate of this oxygen formation reaction fixed whilst investigating the effect of chaning the rate of the triple alpha reaction. They argued that a change of the sub-threshold O-resonance which determines the rate of this reaction by the same amount as the change in C^{12} 0_2^+ would cause the rate of $C^{12} + \varepsilon \rightarrow O^{16} + \gamma$ to change by two orders of magnitude less than that of the triple alpha reaction. In this Section we examine this claim. On first sight it appears odd since the relevant O-resonance is sub-threshold by only 45keV. Since Schlattl et al [2004] consider changes in resonance energy of 100keV, it would seem that the oxygen formation reaction would become resonant. This would be expected to radically increase its rate. If so, the elemental abundance might be far more sensitive to the resonance energies than Schlattl have modelled.

To be completed…use the resonance formulae of Chapter 21 to estimate the rate of $C^{12} + \varepsilon \rightarrow O^{16} + \gamma$ when the resonance is just above threshold.

6. The Implications of Be^8 Stability

In discussing the Hoyle coincidence, one rather obvious factor is generally omitted from consideration. This is that a sufficient change in the strength of the nuclear force would result in beryllium-8 becoming stable. After all, the only reason why the C^{12} 0_2^+ resonance energy is so important is to permit a sufficiently rapid capture of an alpha particle by Be^8 before it decays. As soon as Be^8 becomes stable[1], we have all the time we might want for the subsequent reaction.

Be^8 misses being stable by a mere 91.9keV. The formulae of Chapter 8A can be used to estimate what change in the strength of the nuclear force is required to render Be^8 stable. Based on the data presented in Chapter 8A, η_{Be} is about 4 and η_α/η_{Be} is about 0.9. This results in a fractional change in the Be^8 resonance energy with respect to the two-alpha threshold of -242ε, where ε is the fractional change in the strength of the nuclear force. To make Be^8 stable, the fractional change in its energy with respect to the two-alpha threshold must be -1. Thus, we require $\varepsilon \approx 1/242$ to make Be^8 stable, i.e. an ***increase*** in the strength of the nuclear force of ~0.4%.

This is just the order of magnitude change in the strength of the nuclear force which has been envisaged above, leading to C^{12} 0_2^+ resonance energy changes of the order of 100-200keV (though we cannot be confident of the precise correspondence, see Chapter 8A). According to the results of Schlattl et al [2004], changes of ~100keV might not be biologically fatal. However, the results suggest that changes of ~200 keV or more might well compromise conventional biochemistry due to lack of carbon or oxygen. But we now see that an *increase* in the strength of the nuclear force of this magnitude would make beryllium-8 stable. This would cause a qualitative change in the evolution of stars and their nucleosynthesis of carbon and oxygen.

When a star's hydrogen has converted fully into helium-4 within a certain region, the subsequent burning of helium into carbon via the triple alpha reaction does not occur immediately. Further gravitational contraction is required before the temperature reaches the ignition point for the triple alpha reaction. In a universe with stable Be^8 the end point of hydrogen burning sequences may not be He^4 but Be^8. This will depend upon the ignition temperature of the reaction $He^4 + He^4 \rightarrow Be^8 + \gamma$ in such a universe. If it is within the range of hydrogen burning temperatures (say, around 14

[1] Of course, if Be^8 were only borderline stable, then it would undergo photodisintegration. We are assuming here that its binding energy is many times the prevailing kT.

MK to 30 MK), then hydrogen burning will terminate at Be^8. This seems likely since the usual ppII/ppIII sequences involve the He^3-He^4 reaction $He^3 + He^4 \to Be^7 + \gamma$, so the Coulomb barrier of two double-charged reactants is surmountable at these temperatures. This scenario is particularly likely is Be^8 is only moderately stable, so that its energy lies only a little below the two-alpha threshold. In this case the rate of the reaction $He^4 + He^4 \to Be^8 + \gamma$ will be enhanced by the proximity of the Breit-Wigner peak.

Since $He^4 + He^4 \to Be^8 + \gamma$ does not require temperatures to be elevated above normal hydrogen burning temperatures, it follows that subsequent burning of the beryllium produced is not likely to occur immediately. The reason is that the Coulomb barrier for a $Be^8 + Be^8$ reaction is far higher than the Coulomb barriers involved in the hydrogen burning sequences. The height of the Coulomb barrier is determined by,

$$b = \frac{\pi}{2} Z_a Z_b \alpha \sqrt{2 m_R c^2}\,, \qquad \text{where, } m_R = \frac{M_a M_b}{M_a + M_b}$$

The rate of the reaction is related to this barrier height by,

$$R[T] \propto \exp\{-f_{min}\} \quad \text{where,} \quad \tilde{b} = \frac{b}{\sqrt{kT}} \quad \text{and} \quad f_{min} = 3\tilde{b}^{2/3}$$

Defining $b_0 = \frac{\pi}{2}\alpha\sqrt{2M_p c^2}$, the normalised Coulomb barriers, b/b_0, for several relevant reactions are given in the Table below:-

Reaction	**b/b_0**
$H^1 + H^1$	$\frac{1}{\sqrt{2}} = 0.707$
$He^3 + He^4$	$4\sqrt{\frac{12}{7}} = 5.237$
$He^4 + He^4$	$4\sqrt{2} = 5.657$
$C^{13} + H^1$	$6\sqrt{\frac{13}{14}} = 5.782$
$N^{14} + H^1$	$7\sqrt{\frac{14}{15}} = 6.763$
$N^{15} + H^1$	$7\sqrt{\frac{15}{16}} = 6.778$
$O^{16} + H^1$	$8\sqrt{\frac{16}{17}} = 7.761$
$O^{17} + H^1$	$8\sqrt{\frac{17}{18}} = 7.775$
$Be^8 + Be^8$	32

All the above reactions, bar the last, occur in the pp sequences. Their Coulomb barrier heights are not radically different. However, the $Be^8 + Be^8$ reaction has a far higher Coulomb barrier. Hence further gravitational collapse of the star will be required to raise the temperature before beryllium burning commences in our alternative universe with stable Be^8. What might the ignition temperature of beryllium be?

To estimate this we note that the normalised Coulomb barrier height, b/b_0, for carbon burning, $C^{12} + C^{12}$, is $36\sqrt{6} = 88.18$. Carbon burning ignites at around 500 MK. From the above reaction rate relationship we see that the rate will become significant when the quantity $\frac{b}{b_0\sqrt{T}}$ becomes sufficiently small. Judging from the example of carbon burning, this critical value is ~$88.18/\sqrt{500} = 3.94$ (with T in MK). Hence, the ignition temperature for beryllium burning can be crudely estimated to be ~$(32/3.94)^2 = 66$ MK. This compares with helium burning via the triple alpha reaction in the real universe which occurs at ~100-150 MK.

But what about carbon and oxygen production? Well, in this alternative universe these would be the principle products of beryllium burning, along with that other key biological element, nitrogen. The reactions would most likely be,

$$Be^8 + Be^8 \to O^{16} + \gamma$$

$$Be^8 + Be^8 \to C^{12} + He^4$$

$$Be^8 + Be^8 \to N^{15} + H^1$$

Why should we regard these reactions as likely? Well, it would be foolish to pretend to certainty in such a matter. Once a key parameter of the universe is changed, it is with considerable trepidation that one guesses the outcome. However, since we have *increased* the strength of the nuclear force, it is likely that any reaction which is exothermic in the real universe will also be exothermic in the modified universe. All the above reactions are exothermic, the release energy being 14.624, 7.464 and 2.493 MeV respectively. It is presumably true that all the above reactions are possible in the real universe. The only reason that they are not familiar is that beryllium-8 concentrations never become high enough to drive the reactions, due to the instability of Be^8.

We have not listed $Be^8 + Be^8 \to O^{15} + n$ since this is endothermic in the real universe, by ~1 MeV. However, it is quite possible that this reaction would become exothermic and hence proceed in the alternative universe envisaged.

It is not possible to say anything definitive about the relative abundances of carbon and oxygen (and nitrogen) in this alternative universe. This would require the rates of the above reactions to be quantified. It would also require a detailed knowledge of the stellar physics in the alternative universe. In particular it would be important to determine whether dredge-ups occurred at appropriate times, to provide the mechanism for transportation of the carbon and oxygen to the envelope. We have seen that the third dredge-up is intimately related to thermal instabilities and the phenomenon of the helium flash. Would there be a "beryllium flash" in the alternative universe? Would concentric shells of beryllium and hydrogen burning be established? Would thermal instabilities occur between them? Would the nuclear heating rates undergo transient spikes great enough to drive the convective boundary into the C and O production regions? We cannot pretend to have any real idea of the likelihood of these processes. It may be relevant, however, that the temperature of beryllium burning is only about half that of helium burning in the real universe. This will have a significant impact on the heat transport mechanisms and hence potentially also on the mass transport mechanisms.

The stable-Be^8 universe does appear to have a natural advantage in regard to producing a balance of carbon and oxygen production. This is because both elements are (potentially) being formed in parallel, both being products of a $Be^8 + Be^8$ reaction. It would seem easier to obtain a balanced quantity of both elements via parallel reactions than via the sequential process occurring in the real universe, i.e. carbon formation via the triple alpha reaction followed by a subsequent alpha capture to form oxygen. The latter process carries the inherent threat that the second reaction gob;es up all the carbon formed in the first step.

Indeed, the alternative, stable-Be^8, universe, seems altogether simpler and more obvious than the route that nature has actually chosen. Rather than the universe seeming contrived by a beneficent Creator to ensure an abundance of higher elements, it could whimsically be argued that a conspiracy by the Evil One to frustrate this objective has only narrowly, and rather clumsily, been averted.

7. Conclusions

- Livio et al showed that a C^{12} 0_2^+ resonance energy increase of 60keV had little effect on carbon and oxygen production.
- The early results of Csoto et al, Refs.[12, 13], suggested that C^{12} 0_2^+ resonance energy changes of ±156keV would cause about two orders of magnitude decrease in either carbon or oxygen production (for an increase or a decrease respectively).
- The results of Schlattl et al [2004] imply that, for the dominant contribution due to high mass stars, an increase of 100keV in the resonance energy leads to the carbon ejected into the ISM being reduced by an order of magnitude. Conversely, a decrease of 100keV in the resonance energy leads to the oxygen ejected into the ISM being reduced by between a factor of four and a factor of ten.
- Schlattl et al [2004] regard their results as showing less sensitivity of carbon and oxygen production than their earlier models, Refs.[12, 13].
- However, it could be argued that the results of Schlattl et al [2004] are broadly comparable with Livio et al and Csoto et al, Refs.[12,13], i.e. an intermediate change of resonance energy yields an intermediate change in elemental production.
- However, the more significant issue is that Chapter 8A has shown that resonance energy changes of these magnitudes, up to ±156keV or even greater, would be almost certain to occur as a result of changes in the strength of the nuclear force of a fraction of a percent.
- Consequently, the production of carbon and oxygen in stars would appear to be fine tuned. A change in the strength of the nuclear force of ~1% or less would appear to change the abundance of either carbon or oxygen in the ISM by two or more orders of magnitude.

- However, other than for very small changes in the strength of the nuclear force, this fine-tuning is one-sided. If the change in the nuclear force caused Be^8 to become stable, then a qualitative change in stellar evolution and nucleosynthesis would result. [Note that it is not clear whether an increase or a decrease in the strength of the nuclear force is required to achieve this, though published cluster models suggest am increase is required].
- The stability of Be^8 would lead to a simpler, more straightforward, route for the synthesis of carbon, oxygen and nitrogen than the real universe has adopted. Rather than the universe seeming contrived by a beneficent Creator to ensure an abundance of higher elements, it could whimsically be argued that a conspiracy by the Evil One to frustrate this objective has only narrowly, and rather clumsily, been averted.

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