Chapter 8A – The Hoyle Coincidence Part 1:
Is The $C^{12}[0_2^+]$ Resonance Fine-Tuned?

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TRACK 1

1. Introduction

In Chapter 21 of the Cosmology Tutorial we showed, in a highly simplified manner, how comparable quantities of carbon and oxygen normally result from helium burning in red giant stars. We showed how the key nuclear reaction rates could be estimated from the energy levels of certain nuclear resonances. We also demonstrated, albeit very crudely, that reducing the rate of the triple alpha reaction by a factor of 100 could result in radically reduced amounts of carbon. Similarly, increasing the rate of the triple alpha reaction by a factor of 100 could result in oxygen levels reducing to a few percent of their actual values. Alternatively, changing the rate of the subsequent reaction, the capture of a further alpha particle to form oxygen, could have a similar effect.

In this Chapter the Hoyle Coincidence is described in qualitative terms. The sensitive dependence of the nuclear reaction rates on certain resonance energies is emphasised. We then explore just how sensitive these resonance energies are to the strength of the nuclear force.

In the following Chapter we examine carbon and oxygen production in stars, reviewing results from the literature based on detailed stellar models.
In particular we shall be interested in the change in reaction rates required to reduce carbon or oxygen levels to less than 1% of normal. We shall see that the carbon and oxygen abundances are not as sensitive to the reaction rates as the simple arguments of Chapter 21 suggested. Nevertheless, the sensitivity of the resonance energies to the strength of the nuclear force, and the sensitivity of the reaction rates to these resonance energies, imply that our universe involves a remarkable degree of fortuitousness. However, this fine-tuning is only one-sided.

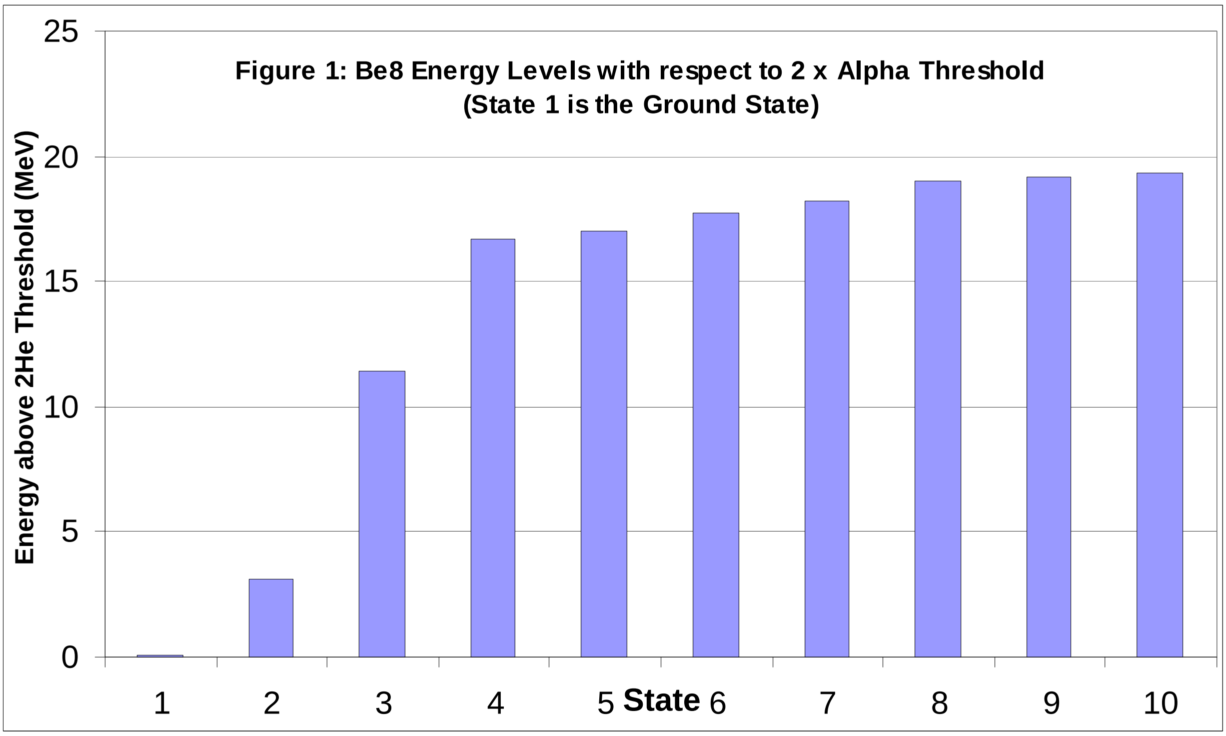
2. The Hoyle Coincidence

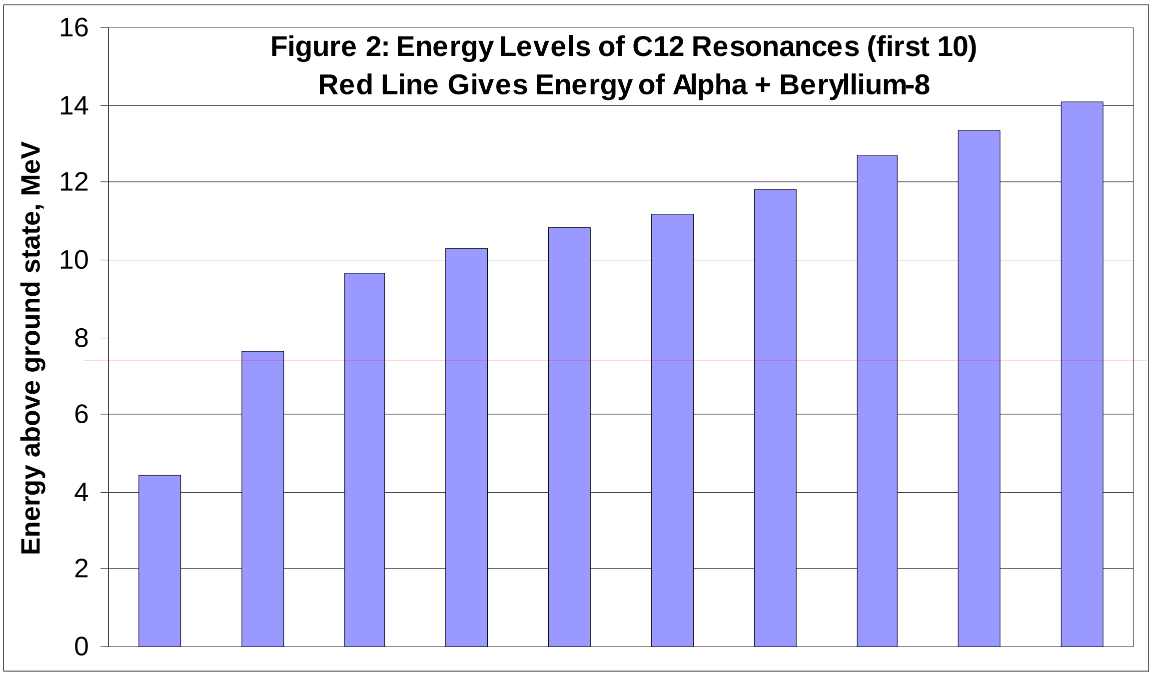
The Hoyle coincidence is claimed to consist of three sequential coincidences. The first is that Beryllium-8 can be formed resonantly from two alpha particles, i.e. its energy is just slightly greater than the two alpha threshold (by 91.9 keV). Because Be^8 is unstable, in order for there to be a significant equilibrium concentration of Be^8 it is necessary that its formation reaction is very fast. The exponential dependence of resonant reaction rates on the resonance energy (above threshold) means that if the energy of Be^8 were (say) twice as great, then the reaction rate would be fatally slower.

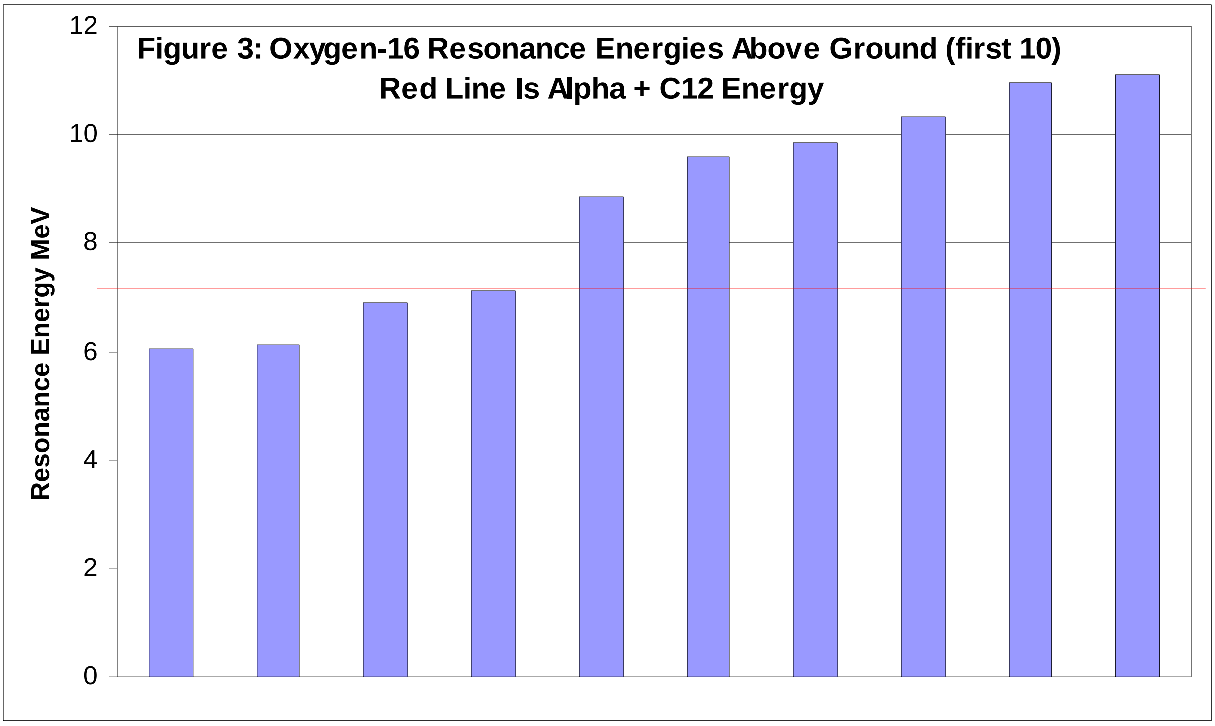
In order to convert a very low Be^8 concentration into a significant carbon concentration, the capture of a further alpha particle also has to be resonant. The second coincidence is that there is indeed a resonance of C^{12} at an appropriate energy to make this reaction resonant. The existence of this resonance was predicted by Hoyle for this reason. It was actively sought and found following his prediction in 1953. This 'missing link' permitted Hoyle to finally solve the stellar nucleosynthesis problem in his classic 1954 paper, Reference [15].

Finally, to avoid all the carbon being immediately cooked into oxygen by a further alpha particle capture, it is necessary that this reaction is *not* resonant. But the rate of alpha particle capture by carbon must result in a balance of carbon and oxygen production. The third coincidence is that the $\alpha + C^{12} \rightarrow O^{16}$ reaction misses a resonance by just 45 keV, fulfilling these conditions.

The coincidences are illustrated by the following energy levels plots:-







In all three cases, the lowest resonances lie at a few MeV above the nuclear ground state.

Consider firstly Figure 1. 'State 1' is the ground state. Its level above the two-alpha threshold is barely discernible on this scale. Contemplate the fact that if this 'almost-zero' energy were twice as big there would be no carbon in the universe and hence no life as we know it. This is the first of the Hoyle coincidences.

Consider now Figure 2. In order for the carbon formation reaction to be resonant, the red line (the $\alpha + {}^8_4Be$ threshold) must be just *below* one of the resonance energies. This is indeed the case. Certainly this seems rather coincidental – but just *how* coincidental? We will see below.

Finally, consider Figure 3. We now need the red line (the $\alpha + C^{12}$ threshold) to be just *above* a resonance energy – and a long way below the next resonance energy, so as to avoid being resonant. The red line does indeed seem curiously well placed to achieve just these conditions. But just how coincidental is it, quantitatively? We shall see below.

The reactions and energy released are as follows:-

$\alpha + \alpha \rightarrow {}^8_4\mathrm{Be}$	Endothermic	$\Delta E =$ -91.9 keV
$\alpha + {}^8_4\mathrm{Be} \rightarrow {}^{12}_6\mathrm{C}^*[0_2^+]$	Endothermic	$\Delta E =$ -287.7 keV
${}^{12}_6\mathrm{C}^*[0_2^+] \rightarrow {}^{12}_6\mathrm{C}[GS]$	Exothermic	$\Delta E =$ 7654.2 keV
$\alpha + {}^{12}_6\mathrm{C} \rightarrow {}^{16}_8\mathrm{O}[1_1^-]$	Exothermic	$\Delta E =$ 45.0 keV
${}^{16}_8\mathrm{O}[1_1^-] \rightarrow {}^{16}_8\mathrm{O}[GS]$	Exothermic	$\Delta E =$ 7116.9 keV
	Net	**$\Delta E =$ 14.4365 MeV**

3. Objection to the Hoyle Coincidence

Before we turn to a quantitative examination of the Hoyle coincidences we note that not all informed parties are impressed. Some think there is no coincidence at all. For example, Weinberg, Reference [1], says, "*I don't set much store by the famous 'coincidence' emphasised by Hoyle, that there is an excited state of C^{12} with just the right energy to allow carbon production via $\alpha - Be^8$ reactions in stars. We know that even-even nuclei have states that are well described as composites of α-particles. One such state is the ground state of Be^8, which is unstable against fission into two alpha particles. The same α-α potential that produces that sort of unstable state in Be^8 could naturally be expected to produce an unstable state in C^{12} that is essentially a composite of three alpha particles, and that therefore appears as a low-energy resonance in $\alpha - Be^8$ reactions. So the existence of this state doesn't seem to me to provide any evidence of fine tuning*."

It is with considerable trepidation that I venture to disagree with Weinberg. The above argument does explain the existence of Be^8 and C^{12} states which are close to the two-alpha and three-alpha thresholds respectively. And the same argument can be applied to O^{16}. Thus, we could expect states of low energy with respect to the Nα threshold for each of Be^8, C^{12} and O^{16}. However, to reproduce the observed state of affairs we require the first two to be resonant, and the third to be non-resonant. Thus, the first two must have small positive energies and the third a small negative energy with respect to the alpha capture thresholds. Perhaps the fact that this is realised is not a particularly remarkable coincidence, i.e. a 1 in 8 chance. However, we will see below that the numerical size of the deviations of these energies from zero (i.e. +91.9, +287.7 and -45.0 keV with respect to the reactant threshold), as well as their signs, are crucial in producing a universe with a balance of carbon and oxygen. Weinberg's argument would appear to suggest that merely being close to zero is enough. It is not.

In passing we note that it is no trivial matter to attempt to make Weinberg's argument quantitative and actually predict the energy levels of Nα nuclei. Such models are known as cluster models. The accuracy with which the energy level of the 0_2^+ resonance of C^{12} has been determined by such models is not impressive. For example, Pichler et al, Reference [2], derive results in the range 640 to 830 keV, compared to the 3α threshold of 379.6 keV. More recently Suzuki et al, Reference [16], have argued that non-local NN forces are required for a good description of these states, but their result of 597keV for the 0_2^+ resonance energy with respect to the 3α threshold is hardly any better. The result of Tursunov, Reference [3], is even further away from reality, i.e. 1550 keV. None of these estimates of the 0_2^+ resonance energy would be consistent with significant carbon production as they stand, the triple alpha reaction rate would be too slow.

We will see in Chapter 8B that a change in the C[0_2^+] resonance energy of ??? keV is enough to reduce either the carbon or the oxygen abundance to ~1% of its actual value. In other words, the change in the resonance energy level required to cause a profound difference in the carbon or oxygen abundance is small compared with the error in cluster model calculations of the energy level. To make this sensitivity clear, note that the triple alpha reaction rate is proportional to the exponential of the $C^{12}\ 0_2^+$ resonance energy with respect to the triple alpha threshold, i.e.,

$$R[T] \propto \exp(-E_R / kT)$$

At a temperature characteristic of a red giant star burning helium, say 150 MK, we have kT = 13 keV. Even the closest of the above cluster model estimates of the resonance energy is adrift by 217 keV, which means that the resulting reaction rate would be slower a factor of exp(-217/13) $\approx 10^{-7}$. Consequently, Weinberg's argument is simply not precise enough to account for the degree of fine tuning which is realised in practice.

The rest of this Chapter is devoted to examining the sensitivity of these key resonance energies to the strength of the nuclear force.

4. How Do We Expect The Resonance Energies To Change?

We face a disadvantage. We need to examine the resonance energies with an accuracy of the order of 100 keV or better, but nuclear theory is just not that accurate as regards the prediction of absolute energies. So, how would we expect the nuclear binding energies to change if the strength of the nuclear force were increased? The simplest analogy is with the energy levels of electrons in atoms. For a one electron atom, or ion, the binding energies of the Schrödinger states are given by,

$$B_n = -E_n = \frac{1}{n^2} \cdot \frac{\alpha^2 Z^2 mc^2}{2}$$

Thus, the Schrödinger state binding energies are simply proportional to the square of the "strength" of the electromagnetic interaction, which is specified by the fine structure constant, α, in this case.

Introducing relativistic effects via the Dirac equation breaks some of the degeneracies and leads to a refinement of the energy levels, which now depend upon the total angular momentum quantum number, j, as well as the principle quantum number, n, given by,

$$B_n = mc^2 - E = mc^2 - mc^2 \left[1 + \left(\frac{Z\alpha}{n - j - 0.5 + \sqrt{(j+0.5)^2 - Z^2\alpha^2}} \right)^2 \right]^{-1/2}$$

Thus, the binding energies are no longer exactly proportional to α^2. However, the bracket can be expanded in powers of α^2, and the leading term is proportional to α^2 (since the mc^2 terms cancel) and subsequent terms are very small in comparison. Thus, the binding energy is again proportional to α^2 to a very good approximation. This works only because $\alpha \ll 1$. So, in the case of the strong nuclear force, for which the effective low energy dimensionless coupling constant is greater than unity, we can guess we might be in trouble with such simple ideas.

That we are indeed in trouble is confirmed by considering that old stand-by of physicists, the square well potential. As a model of nuclear states, a square well potential is entirely hopeless once one considers more than two nucleons. Nevertheless it is a useful pedagogic illustration…..Need to develop my knowledge of the square well energy levels and then include something here if it's worthwhile.

In the light of the above difficulty, we can reasonably ask why the simple scaling of the binding energies as α^2 works in the case of the atomic electron states. The reason is that, in this case, the kinetic energy happens also to be proportional to α^2. This is a 'fluke' which occurs in the case of power-law forces. Considering classical circular motion and balancing the forces for a force law $F = \alpha / r^n$ gives,

$$\frac{mv^2}{r} = \frac{\alpha}{r^n}$$

But the potential energy from which this force law could be derived is $V = -\frac{\alpha}{(n-1)r^{n-1}}$, consistent with $F = -\frac{\partial V}{\partial r}$. So that the kinetic energy is related to the potential energy by,

$$KE = \frac{mv^2}{2} = \frac{\alpha}{2r^{n-1}} = -\frac{(n-1)}{2} PE$$

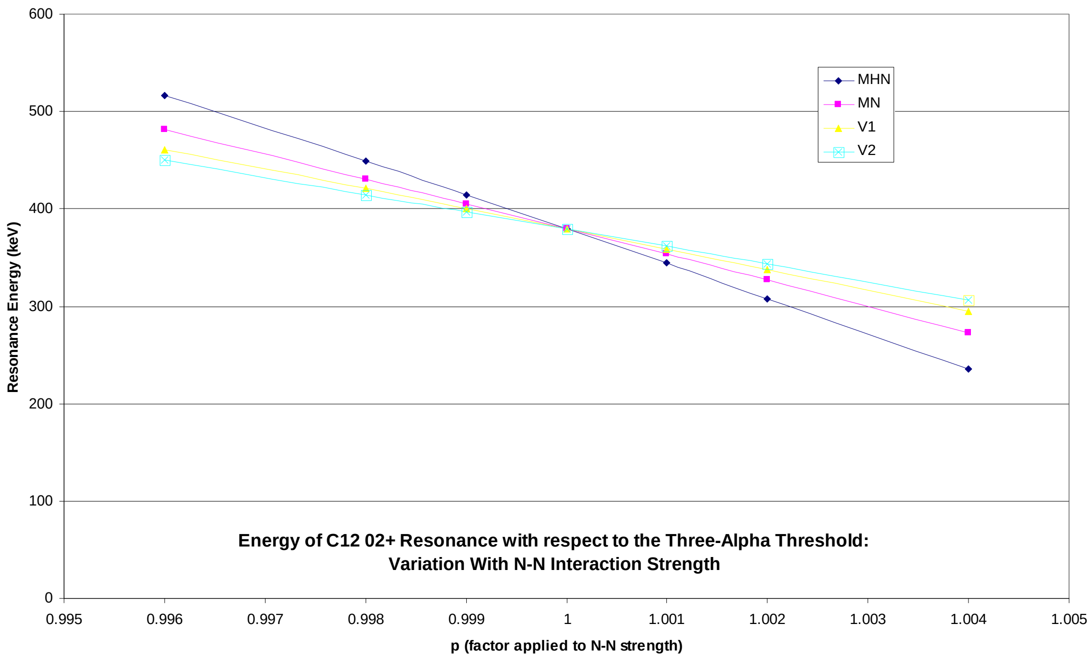
Hence, the kinetic energy is just a numerical constant times the potential energy, and hence both will vary proportionally if the strength of the force is changed. (We have conveniently ignored the fact that stable orbits do not exist for $n \neq 2$). For the familiar case of an inverse square law force, n = 2, this reproduces the Virial Theorem result that KE = -0.5PE, and hence that the binding energy, $|PE + KE|$ is equal to the kinetic energy, both being $\frac{\alpha}{2r}$. In the quantum mechanical equivalent, the size of the orbit, r, is inversely proportional to the strength of the force, α. Consequently, the KE and the PE are both proportional to α^2, as given by the Schrödinger energy levels. But this is a fluke attributable to a simple, single parameter, power-law force.

This issue is crucial. If the nuclear force behaved in this manner, then all energy levels would be proportional to the same function of its strength (which we would then suppose to be captured by a single parameter, call it $\alpha_s \propto g_s^2$). In analogy with atomic energy levels, suppose the nuclear levels were proportional to $\alpha_s^2 \propto g_s^4$. It would then follow that the energy of the C^{12} 0_2^+ resonance and the energy of the alpha particle would both scale as $\alpha_s^2 \propto g_s^4$. Consequently, the energy of the C^{12} 0_2^+ resonance with respect to the three alpha threshold would also scale as $\alpha_s^2 \propto g_s^4$. Let us say that we could tolerate a change in this energy with respect to the reaction threshold of ±150 keV, which is ±40%. We could then tolerate a variation in the strength of the nuclear force, as measured by α_s, of ±18%, or of ±9% in g_s. This is of similar magnitude to the decrease in the strength of the nuclear force which would unbind the deuteron. Thus, if this simple scaling were correct, the Hoyle coincidence would add nothing more to the fine-tuning of the strong force.

The message, however, is that such a simple scaling behaviour is not to be expected for a multi-component force such as the nuclear force, especially for binding energies which are substantial fractions of the particle rest masses. Much greater sensitivities of the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold have been claimed in the literature, with variations of roughly ±150 keV arising from only ±0.4% changes in the strength of the nuclear force.

5. Results of Csoto, Oberhummer and Schlattl

Given that simple scaling arguments do not apply, what do existing models of the C^{12} resonances suggest may be their sensitivity to the strength of the nuclear force? Csoto, Oberhummer and Schlattl, References [12] and [14], have used the cluster model approach to derive the energy levels of resonances of C^{12} from various assumed N-N force models. Whilst the absolute prediction of the 0_2^+ resonance level is less than impressive, as noted already, these studies nevertheless provide a means of deducing the change in the resonance energy for a given percentage change in the underlying nucleon-nucleon strong interaction. Csoto et al find that a ±0.4% change in the N-N force can produce up to a ±38% change in the 0_2^+ resonance energy with respect to the three-alpha threshold (depending upon what force model is used). These authors claim that the great sensitivity of this energy level to a far smaller change in the N-N force is an example of non-linear amplification (Reference [14]). Their results are summarised in the following graph:-



The four lines are for four different underlying N-N force models. On the strength of the results of Csoto et al, the nuclear force would appear to be fine tuned to a fraction of a percent.

Lamentably, however, it is likely that these results are subject to overwhelming numerical error. The reason is that the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold is found by calculating separately the absolute binding energy of the C^{12} 0_2^+ resonance and the absolute binding energy of three alpha nuclei. In this universe these quantities are 84,508 keV and 3 x 28,296 = 84,887 keV. The difference between these two figures is the usual C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold, i.e. 379 keV. Csoto et al's estimate of the perturbed C^{12} 0_2^+ resonance energy with respect to the perturbed three-alpha threshold therefore depends upon a delicate cancellation between two large numbers. If the fractional change in the C^{12} 0_2^+ resonance binding energy is δ, and the fractional change in the alpha binding energy is ε, the perturbed C^{12} 0_2^+ energy with respect to the perturbed reaction threshold can be written,

$$84{,}887(1 + \varepsilon) - 84{,}508(1 + \delta) = 379 + (84{,}887\varepsilon - 84{,}508\delta) \quad \text{keV}$$

Csoto et al's results imply that, for a 0.4% change in the nuclear force, the quantity $(84{,}887\varepsilon - 84{,}508\delta)$ is 144 keV. If $\delta = \varepsilon$ this would imply that $\delta = \varepsilon = 0.38$. This is what Csoto et al mean by a "non-linear amplifier", Reference [14], since an input change of only 0.4% (i.e. a factor of 0.004) produces an output response of a change by 38% (a factor of 0.38), an apparent gain of x95.

However, the fractional changes, δ and ε , in the energies of the carbon resonance and the alpha particle are unlikely to be the same. A change of 144 keV in the C^{12} 0_2^+ energy with respect to the perturbed reaction threshold can come about for fractional changes in the C^{12} 0_2^+ energy and the alpha particle energy which are of the same order as the fractional change in the strength of the nuclear force. All we require is to exploit the delicate cancellation involved in $(84{,}887\varepsilon - 84{,}508\delta)$. Thus, if we take $\delta = 0.00231$ and $\varepsilon = 0.004$, the we reproduce Csoto et al's result that

$$(84{,}887\varepsilon - 84{,}508\delta) = 144 \text{ keV}$$

Consequently, it is not necessary for the nuclear energies to be especially sensitive to changes in the nuclear force in order tom reproduce Csoto et al's results. They can be reproduced by nuclear energy changes which of the same fractional order as that of the nuclear force strength. The key point is that the C^{12} resonance energy ***with respect to the perturbed reaction threshold*** is the result of a delicate cancellation and can therefore change by a much larger percentage.

It is difficult to have great confidence in Csosto et al's numerical result that a 0.4% change in the nuclear force produces a change of 144 keV in the C^{12} 0_2^+ energy with respect to the perturbed reaction threshold. This is because the error in reproducing the absolute binding energies of nuclear states with such models is, at best, of the order of MeV. To reproduce the difference between two such energies to an accuracy of better than ~100 keV therefore seems unlikely. On the other hand, this should not detract from the main message of their work. This is that the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold is likely to be very sensitive to changes in the strength of the nuclear force. The reason is that the fractional change in this energy,

$$\xi = \frac{\delta \Delta E}{\Delta E} = \frac{84{,}887\varepsilon - 84{,}508\delta}{379}$$

depends upon a delicate cancellation. Even though δ and ε may be of the same order as the fractional change in the strength of the nuclear force, they only have to be slightly different to result in ξ being a far larger fraction.

In the next Section we investigate how the magnitude of ξ might be found.

6. First Order Perturbation Theory

The question as to how energy levels change for a small change in the strength of the interaction is the archetypal perturbation theory question. Suppose the unperturbed Hamiltonian is written as H = K + V, where K is the kinetic energy operator and V is the sum of all interaction terms between the nucleons. The form of the kinetic energy operator is not required for our purposes, but for completeness we note that it is,

$$K = -\frac{\hbar^2}{2} \sum_{i=1}^{A} \frac{1}{m_i^2} \cdot \frac{\partial^2}{\partial x_i^2}$$

The sum is over all the nucleons in the nucleus. If the unperturbed state of the C^{12} 0_2^+ resonance is written $|u\rangle$, then the binding energy of this state is,

$$B_u = -E = -\langle u|H|u\rangle = -\langle u|(K+V)|u\rangle = -(K_u + V_u)$$

where K_u and V_u are the expectation values of the total kinetic energy and total interaction energy in the unperturbed state. For a bound state, V_u is negative and larger in magnitude than the positive kinetic energy, K_u, so that the energy of the state, E = K + V, is negative and the binding energy, B_u = -E, is positive.

We now consider a small fractional change, ε, in the total interaction Hamiltonian. The Hamiltonian is now H = K + (1+ε)V. According to first order perturbation theory, the change in the energy of a given state is approximately given by the expectation value of the change in the interaction Hamiltonian, εV, with respect to the unperturbed state,

$$\delta E \approx \langle u|\varepsilon V|u\rangle = \varepsilon V_u \quad \text{or} \quad \delta B \approx -\varepsilon V_u$$

We introduce the notation: $\eta \equiv \frac{V_u}{E}$, which expresses the interaction energy as a multiple of the energy of the state. Both V_u and E are negative, so η is positive. Also, $|V| > |E|$ so that η > 1. If the kinetic energy, K, and $|V|$ are close in magnitude, then η will be a lot bigger than unity. Hence,

$$\delta E \approx \varepsilon \eta E$$

Thus, the fractional change in the binding energy of a nuclear state is a factor η larger than the fractional change, ε, in the strength of the nuclear force which causes it. We shall see shortly what the likely magnitude of η might be.

The above reasoning applies equally to the resonances of carbon and the alpha particle. However, the value of η, which depends upon the relative proportions of kinetic and interaction energies, will in general differ between different nuclei. Suppose we write the energy of the C^{12} 0_2^+ resonance with respect to the three-alpha threshold as,

$$\Delta E = E_C - 3E_\alpha$$

Then the change in this energy due to a change in the strength of the nuclear force is,

$$\begin{aligned}\delta\Delta E = \delta E_C - 3\delta E_\alpha &= \varepsilon[\eta_C E_C - 3\eta_\alpha E_\alpha] = \varepsilon[-84{,}508\eta_C + 84{,}887\eta_\alpha] \\ &= 84{,}508\varepsilon\eta_C\left[1.00448\frac{\eta_\alpha}{\eta_C} - 1\right] \quad \text{keV}\end{aligned}$$

Dividing by the value of $\Delta E = E_C - 3E_\alpha$ in this universe (379 keV) gives the fractional change in this quantity,

$$\frac{\delta\Delta E}{\Delta E} = 223\varepsilon\eta_C\left[1.00448\frac{\eta_\alpha}{\eta_C} - 1\right]$$

Thus, if η_α and η_C were precisely equal, this would reduce to $\frac{\delta\Delta E}{\Delta E} = \varepsilon\eta_C$. In this case, whilst the fractional change in the resonance energy with respect to the reaction threshold would indeed exceed the fractional increase in the strength of the nuclear force, it does so only by the factor η_C. We shall see below that this is likely to lie between 2 and 5. This is an enhancement, but not a dramatic enhancement.

However, it is very unlikely that η_α and η_C will be equal to the precision required for this to be true. And very slight deviations from equality of η_α and η_C will cause $\frac{\delta\Delta E}{\Delta E}$ to be dramatically greater than the fractional change in the strength of the nuclear force, ε. For example, if η_α were just 9% smaller than η_C we would get $\frac{\delta\Delta E}{\Delta E} \approx -19\varepsilon\eta_C$. For η_C in the range 2 to 5 this implies that a 0.4% *decrease* in the strength of the nuclear force would produce an *increase*, $\frac{\delta\Delta E}{\Delta E}$, in the range 15% to 38%. The latter reproduces Csoto et al’s results (see Section 5).

Consequently, Csoto et al’s results appear very credible, although the precision of their result is in doubt due to the considerable uncertainty in the value of η_α / η_C.

However, in view of the possible range of values of this ratio discussed below, the sensitivity of the triple alpha reaction to the strong force is likely, if anything, to be even greater than their result suggests. The "gain" of the amplifier is

$$\text{Gain} = 223\eta_C\left[1.00448\frac{\eta_\alpha}{\eta_C} - 1\right]$$

This factor becomes large for very modest departures of η_α / η_C from unity. In as far as there is no obvious reason to expect η_α and η_C to be equal to high precision, it follows that the rate of the triple alpha reaction is almost inevitably going to be extremely sensitive to very small changes in the strength of the nuclear force.

7. Sensitivity of the Beryllium Formation Reaction

Although the formation of carbon is often expressed as proceeding via a "triple alpha" reaction, it truth this is merely a shorthand for a succession of three reactions. The first reaction is the resonant formation of unstable Be^8. The second reaction is the resonant formation of the unstable 0_2^+ resonance of C^{12}. Then lastly this resonance decays into the ground state of C^{12}. It is therefore of interest to examine to what degree the first of these reactions, the formation of beryllium, is affected by a change in the strength of the nuclear force. The energy of the ground state of Be^8 above the two-alpha threshold is,

$$\Delta E' = E_{B_e} - 2E_\alpha$$

The change in this energy due to a change in the strength of the nuclear force is,

$$\delta\Delta E' = \delta E_{B_e} - 2\delta E_\alpha = \varepsilon\left[\eta_B E_{B_e} - 2\eta_\alpha E_\alpha\right] = \varepsilon\left[-56{,}499.5\eta_{B_e} + 56{,}591.4\eta_\alpha\right]$$

$$= 56{,}499\varepsilon\eta_{B_e}\left[1.001626\frac{\eta_\alpha}{\eta_{B_e}} - 1\right] \quad \text{keV}$$

Dividing by the value of $\Delta E' = E_{B_e} - 2E_\alpha$ in this universe (91.9 keV) gives the fractional change in this quantity,

$$\frac{\delta\Delta E'}{\Delta E'} = 615\varepsilon\eta_{B_e}\left[1.001626\frac{\eta_\alpha}{\eta_{B_e}} - 1\right]$$

8. The Values of the Ratios η_α / η_C and η_α / η_{B_e}

In this Section we review what values for the ratios η_α / η_C and η_α / η_{B_e} are suggested by nuclear models in the literature. We are interested in the ground states of He^4 and Be^8 and the 0_2^+ resonance of C^{12}. Models of resonances are less common than models of ground states, so we shall also consider the value of η_C for the ground state of C^{12}. The available sources are considered in turn:-

8.1 Buendia, Galvez, Praena and Sarsa [Ref. 17]
Total energies, kinetic energies and total 'potential' energies are given in Table 2 and Figure 3 of Ref.[17] for the ground states of He^4, Be^8 and C^{12} for several different N-N force models. These give,

Model	K (MeV)	V (MeV)	E (MeV)	η = V/E
		C^{12} G.S.		
BB1 J-HO	202	-303	-101 (-92)	3.0 (3.3)
BB1 J-MB	233	-345	-112 (-92)	3.1 (3.8)
MS3 J-HO	237	-303	-66 (-92)	4.6 (3.3)
MS3 J-MB	218	-291	-73 (-92)	4.0 (3.2)
		Be^8 G.S.		
BB1 J-HO	134	-198	-64 (-56)	3.1 (3.5)
BB1 J-MB	139	-213	-74 (-56)	2.9 (3.8)
MS3 J-HO	134	-177	-43 (-56)	4.1 (3.2)
MS3 J-MB	143	-194	-51 (-56)	3.8 (3.5)
		He^4 G.S.		
BB1 J-HO	61	-99	-38 (-28)	2.6 (3.5)
MS3 J-HO	61	-88	-27 (-28)	3.3 (3.1)

I have emailed for more detailed numerical data (the above were scaled off a very small graph). I have also asked for kinetic energies of the 0_2^+ resonance of C^{12}, since that was also modelled in Ref.[17], but the KE not quoted.

The figures in brackets are the actual total energies and the resulting V/E ratio (assuming the modelled value for V).

8.2 Pieper, Pandharipande, Wiringa and Carlson [Refs.18-20]
Monte Carlo methods have been used by authors Pieper, Pandharipande, Wiringa and Carlson in various permutations, Refs.[18-20], to model light nuclei up to A = 8. Data is provided for the kinetic energy component of the total energy for the ground states. Their results for several force models are given below. The energies do not exactly respect the requirement that K + V = E, as noted by the authors.

Model	K (MeV)	V (MeV)	E (MeV)	η = V/E
		Be^8 G.S.		
AV18	238	-299	-46 (-56)	6.5 (5.3)
AV18/UIX	257	-312	-54	5.8
AV18/IL2	267	-322	-57	5.6
Ref.[20]	256	-324	-68 (-56)	4.8 (5.8)
		He^4 G.S.		
AV18	101	-131	-24 (-28)	5.5 (4.7)
AV18/UIX	114	-142	-28	5.1
AV18/IL2	116	-145	-28	5.2
Ref.[20]	115	-144	-29	5.0

The figures in brackets are the actual total energies and the resulting V/E ratio (assuming the modelled value for V). These estimates of η are substantially larger than those of Buendia et al, above.

8.3 Fujiwara et al, Ref.[21]

Fujiwara et al, Ref.[21], have modelled the ground state of C^{12}. Their results are expressed with respect to the three-alpha threshold. They have been adjusted here by adding three times the alpha binding energy to their quote kinetic energy, and subtracting the same from their total energy. Note that this leads to *twice* this quantity being algebraically subtracted from their potential energy. This is rather a wild guess – could very well be wrong! The results for their three N-N force models are:-

Model	K (MeV)	V (MeV)	E (MeV)	$\eta = V/E$
		C^{12} G.S.		
VN1	160.3	-250.5	-90.3	2.8
VN2	153.8	-244.7	-90.9	2.7
MN	185.0	-279.4	-94.5	3.0

These estimates of η are substantially smaller than those of Buendia et al, above.

In view of the above results, we conclude that there is substantial uncertainty in the values of η. For the ground state of C^{12} results for η range from 2.7 to 4.6, but larger values would likely results from the methods of Pieper et al. For the ground state of Be^8 results for η range from 2.9 to 6.5. For the alpha particle, results range from 2.6 to 5.5.

The uncertainty in η would therefore appear to be crudely around ±50%.

Recall that the equation derived in Section 6 for the fractional change in the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold required that the ratio η_α / η_C be equal to unity to within a small fraction of a percent if extreme sensitivity to the strength of the nuclear force were to be avoided. For example, if η_α were just 0.5% less than η_C, this energy would decrease rather than increase when the nuclear force were made stronger. Thus, even a 100-fold increase in the accuracy of modelling nuclear energies would not suffice to determine even the sign of the energy change, let alone its magnitude with any precision. There seems no prospect in the foreseeable future of nuclear modelling advancing to such precision as to allow a determination of the η values with the precision required to make a definitive statement about the change in the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold. The effect of changes in the strength of the nuclear force on the triple-alpha reaction rate is therefore undecidable at present.

However, these observations do demonstrate one thing clearly. The change in the C^{12} 0_2^+ resonance energy with respect to the three-alpha threshold is almost certainty extremely sensitive to changes in the strength of the nuclear force. This follows because, to avoid such sensitivity, the ratio η_α / η_C must equal unit to within a small fraction of a percent. There is no obvious reason why this should be the case, and the spread in modelling results argues strongly that it certainly is not the case. Hence, whilst we can say essentially nothing about how the triple-alpha reaction rate would change if the strength of the nuclear force were changed, we *can* say with confidence that it would almost certainly change radically.

The implications of the sensitivity of the triple alpha reaction rate for carbon and oxygen formation in stars will be addressed in the second part of this Chapter.

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