New Chapter 3
The Universal Constants

§3.1 Our Set of Universal Constants

The ten dimensionless universal constants to be used here have already been listed at the beginning of §1.1. In this chapter we describe what these constants are. The set of universal constants adopted depends upon the most fundamental physical theory considered. Our ambitions are modest. The history of the universe will be considered only after the first millisecond. Attention can then be restricted to what can be derived using relatively simple, low energy, theory[1]. Quantum field theory (QFT) will be avoided.

Unfortunately this means that the true basis of the standard model of particle physics, which is a gauge field theory, will not be addressed. Compensation for this takes two forms. Firstly, the particle zoo upon which the standard model is based is described without mathematics in Appendix B. Secondly, the 26 universal constants which occur in the standard model will be described in this chapter. Most important are the three so-called 'coupling constants' which define the strengths of the electroweak and strong forces. These also occur in our reduced set of 10 constants.

Providing a quantitative definition of the coupling constants really does require a mathematical setting. In lieu of the full QFT formalism, an explanation of the coupling constants is offered in terms of the energies of interacting particles (see Appendices C, D, E and F). Whilst imprecise, it is hoped that this provides an intuitive feel for what these coupling constants mean. Finally, in the interests of honesty, it is not possible to ignore one particular implication of the standard QFT model entirely. This is that the coupling constants are not truly constant but vary with energy. They are referred to as "running coupling constants". This is also described in simple physical terms, in Appendix G. However, in the low energy regime which is required to describe the universe after the first millisecond, the running of the coupling constants will be unimportant in practice.

The disadvantage of starting the clock at about one millisecond is that certain properties of the universe which are imprinted upon it before this time will appear instead to be initial conditions. This is not a disadvantage in practice because, as theory stands at present, consideration of the physics at earlier times does not lead to any reduction in the number of free parameters (though it may do eventually). An exception is the resolution of the flatness and horizon problems, but these will be addressed in chapter ? The other disadvantage of missing out the first millisecond is that we loose the opportunity to discuss the origin of the universe. What happened at time zero? Compensation for this is provided by a highly simplified summary of current cosmological ideas in Appendix A.

The main advantage of starting the clock at around a millisecond is that the physics is really quite simple at that time, and for some time to come thereafter. In contrast,

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[1] That is, the theory is simple, not necessarily the application. Stars and galaxies are as complicated as the depth with which you wish to understand them. Even simple theories produce very complicated applications. Recall that the Newtonian 3-body problem is not analytically soluble, and that it is a challenge to understand a boiling saucepan.

during the first millisecond the physics is complex. Either large numbers of strongly interacting particles are present or irreducibly quantum field theoretic phase changes induce qualitative changes in the nature of the content of the universe. In any case, it appears that we need *fewer* universal constants to describe the universe after the first millisecond than before, at least as theory currently stands. So it is arguably the optimal time to start.

In this book we shall adopt a set of just 13 fundamental constants, which comprises the 10 dimensionless constants listed in §1.1 plus three dimensionful constants. The latter are really just equivalent to defining a set of units, so it is the set of 10 dimensionless constants that matters. This is explained further below. There are nine constants from physics, as follows:-

[1] The speed of light in a vacuum, c.

[2] Planck's constant, $\hbar$. (Strictly, this is Planck's constant divided by 2π, but it will be referred to simply as Planck's constant to avoid unnecessary repetition).

[3] Newton's universal gravitational constant, G.

[4] The electric charge of the proton (equal to minus that of the electron), e.

[5] Fermi's weak nuclear coupling constant, G_F.

[6] The low energy effective coupling constant for the strong nuclear force, g_s.

[7] The mass of the neutron (M_n).

[8] The mass of the proton (M_p).

[9] The mass of the electron (m_e).

The four coupling constants, G, e, G_F and g_s, are described in greater detail in Appendices C, D, E and F. In addition, there are the four cosmological parameters which define the large scale structure of the universe and which have already been introduced in chapter 2:-

[10] The ratio, ξ, of the number of nucleons to the number of photons in the universe. (It may not be obvious, but this ratio is essentially constant).

[11] The average density of dark matter (Ω_d as a fraction of the critical density);

[12] The average density of dark energy (Ω_λ as a fraction of the critical density);

[13] A parameter Q representing the deviation of the primordial universe from perfect homogeneity (and isotropy).

It is possible that developments in theoretical understanding will eventually lead to these last four cosmological constants being derivable from microscopic physics.

The numerical values of all 13 constants are given in Table 3.1, in MKSA units and in Planck units and in particle physicist's "MeV" units where relevant. The 98% confidence limit errors (2 standard deviations) in the last digit(s) are shown in brackets. Seven of the nine physical constants are known to high precision. The strong coupling is only roughly defined. This is not because of imprecision in experimental data involving the strong force, but because the description of the low energy strong force in terms of a single parameter is theoretically imprecise. Even at higher energies a unique coupling constant is not defined, since the coupling is "running", i.e. energy dependent. Excepting g_s, the least well known of the physical constants is G.

The four cosmological parameters are far less precisely known than the physical constants, although they are a very great deal better defined now than a decade ago, thanks to COBE and WMAP [see for example Spergel et al (2006)]. The critical density in the current epoch is ~10^{-26} kgm^{-3} (equivalent to ~6 H atoms per m^3). As a fraction of the critical density, the amount of ordinary (baryonic) matter is $\Omega_b = 0.042 \pm 0.0015$. The average density of cold dark matter plus neutrinos as a fraction of the critical density is $\Omega_{cdm+\nu} = 0.218 \pm 0.024$, and the dark energy density parameter is $\Omega_\Lambda = 0.74 \pm 0.04$. The total density parameter ($\Omega_b + \Omega_{cdm+\nu} + \Omega_\Lambda$) is consistent with unity to within ~1%, and hence consistent with a spatially flat universe. The photon:baryon ratio (ξ) is ~1.6 x 10^9. The so-called "scalar fluctuation amplitude, Q", i.e. the fractional deviation of the cosmic microwave background (CMB) from homogeneity, ~ 2 x 10^{-5}. The fact that all four quantities, $\Omega_{cdm+\nu}$, Ω_Λ, ξ and Q are known to within about 10%, or better, is a triumph of observational cosmology over the last ten years.

§3.2 The 10 Dimensionless Constants

Of the 13 universal constants, three may be chosen to normalise the rest and make them dimensionless. Three are required, of course, to replace the three dimensions of mass, length and time. Physicists generally choose c and $\hbar$ as two of these normalising constants. In particle physics this results in everything being measured in a suitable power of MeV. Normalising to dimensionless quantities requires a third parameter to set a scale for mass or energy. One choice is Planck units, in which the third constant chosen for normalisation purposes is the gravitational constant, G. The resulting Planck units are,

Planck Units

Quantity	Definition	Value (MKS)
Planck length, L_P	$\sqrt{\frac{\hbar G}{c^3}}$	1.6162×10^{-35} m
Planck time, t_P	$\sqrt{\frac{\hbar G}{c^5}}$	5.3911×10^{-44} s
Planck mass, M_{Planck}	$\sqrt{\frac{\hbar c}{G}}$	2.1765×10^{-8} kg
Planck temperature, T_P	$\frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}}$	1.4168×10^{32} K

Planck energy, E_P	$\sqrt{\frac{\hbar c^5}{G}}$	1.2209×10^{19} GeV 1.9561×10^{9} J
Planck density, ρ_P	$\frac{c^5}{\hbar G^2}$	5.15749×10^{96} kg/m^3

In terms of Planck units, our first three universal constants (c, $\hbar$ and G) would be defined as unity, leaving 10 constants in our list – now in dimensionless form. The choice of Planck units is generally regarded as most fundamental. For example, the Planck length and time may effectively define the quanta of spacetime, or at least the size scale at which spacetime ceases to behave as a classical continuum. However, even if the Planck scale is indeed truly fundamental, an alternative mass scale may be more convenient. There is, in any case, a practical reason why metrologists would not wish to adopt Planck units. Planck units fail to do justice to the precision with which many of the universal constants are known. The reason is that the gravitational constant, G, is amongst the least well known of the constants. By adopting Planck units, the uncertainty in G is communicated to all the other constants.

The normalising mass scale to be used here is the proton mass, M_p. Hence, c, $\hbar$ and M_p drop out of our set of constants, but we retain G, now in its dimensionless form as $\alpha_G = \frac{GM_P^2}{\hbar c}$. Consequently our list of 13 universal constants is really only 10 dimensionless constants. The numerical values of the 10 dimensionless constants are given in the final column of Table 3.1.

§3.3 Parameter Space and What Varying a Constant Means

If the dimensionful constants, c, $\hbar$ or M_p, were varied whilst holding all the dimensionless constants fixed, would there be any change in the physical world? The answer is, "no", but this is a matter which can cause confusion. The crucial requirement is that ***all*** the dimensionless constants remain unchanged. In that case, a change in the numerical magnitude of c, $\hbar$ or M_p amounts merely to a change in the units in which c, $\hbar$ or M_p are being measured. There is no change in the physical world.

For example, the numerical magnitude of c may be doubled simply by taking a hacksaw to the standard metre in Paris and cutting it in half. All lengths would be numerically doubled in terms of this (halved) standard length. So $\hbar$ would increase by times 4, for example. In general, quantities with dimensions $M^aL^bT^c$ would change by a factor 2^b. So, the quantum of charge, e, would increase by 2√2. Hence, the dimensionless fine structure constant, $\alpha = \frac{e^2}{4\pi\hbar c}$, would be unchanged. Inevitably, all dimensionless constants will be unchanged because their overall 'b' is zero. Barrow (2003) has expressed this as follows:-

"*The last important lesson we learn from the way that pure numbers like* α *define the world is what it really means for worlds to be different. The pure number that we call the fine structure constant and denote by* α *is a combination of the electron charge, e, the speed of light, c, and Planck's constant,* $\hbar$. *At first sight we might be tempted to think that a world in which the speed of light was slower would be a different world.*

But this would be a mistake. If e, c and ħ were all changed so that the values they have in metric (or any other) units were different when we look them up in our tables of physical constants, but the value of α remained the same, this new world would be observationally indistinguishable from our world. The only things that count in the definition of the world are the values of the dimensionless constants of Nature. If all the masses are doubled in value you cannot tell because all the pure numbers defined by the ratios of any pair of masses are unchanged."

This is clearly true in any example where we change one or more of the definitions of 'metre', 'second' or 'kilogram'. All dimensionful quantities then change in such a manner that, necessarily, the dimensionless quantities are unchanged – and the world itself is unchanged. However, conceptual difficulty can arise if a change is specified as, say, "double the speed of light" without also specifying how all the other constants are to change. Magueijo (2003), Chapter 10, describes very nicely the confusion that can ensue. The whole point of the work that Magueijo (2003) was undertaking was based upon the postulate that the speed of light might have been different in the past. But did we not just claim that a change in the dimensionful parameters makes no difference to the world? No! A change in the dimensionful parameters makes no difference to the world *as long as none of the dimensionless constants change*. In Magueijo's case, α was changing, and this had to be attributed to a change in at least one of e, c and ħ - and c was the chosen culprit in this theory.

Let us suppose that we consider a change in c whilst holding the other 12 constants as listed above fixed (and noting that all nine of the physical constants are dimensionful). What difference does this make to the world? The key to this question is to consider which of the dimensionless constants is affected by the change in c. There are only three. They are $\alpha_G = \frac{GM_P^2}{\hbar c}$, $\alpha_w^p = \frac{G_F M_p^2 c}{\hbar^3}$ and $\alpha = \frac{e^2}{4\pi\hbar c}$. Thus, if c is doubled, holding the other 12 constants fixed, the strength of gravity and the strength of the electromagnetic force are both halved, whereas the strength of the weak nuclear force is doubled. On the other hand, the strong nuclear force, which depends upon $\alpha_s = \frac{g_s^2}{4\pi}$, is unchanged – as are all the other dimensionless constants.

Could the change in c be neutralised as regards its impact on the world by considering appropriate changes to other quantities? Clearly, the answer is "yes", as we have already seen. For example, attributing the change in c to an underlying change in the units of length and/or time will accomplish this automatically. Another way of achieving this is to require that ħ halves when c doubles, and that the Fermi constant, G_F, reduces by a factor of 1/16. This also results in all the dimensionless constants remaining invariant, as does the world.

Is there something fundamental about the fact that gravity, electromagnetism and the weak nuclear force change if c changes, but the strong nuclear force does not? No. This is merely a consequence of the choice of constants. For example, we could employ a new definition of the gravitational constant, such as $G' = G/c$. Adopting G' instead of G now leads to gravity being invariant when c changes. But this is only for the trivial reason that G', being one of our set of constants, is being held fixed by fiat.

The moral is that we should never talk of “varying constant X” but rather of “varying constant X whilst holding the set of constants $\{Y_i\}$ fixed”. It is just as important to define the invariant set $\{Y_i\}$ as it is to define X. Otherwise the actual nature of the change envisaged has not been defined at all. However, this is not the most general type of change that may be considered.

The correct way to think about changes in the constants is to consider the whole N-dimensional parameter space (where N = 10 for our reduced set of dimensionless constants). A variation consists of defining a vector in this space: $\delta\overline{C} = \overline{C}' - \overline{C}$, where $\overline{C}$ represents the N-dimensional set of constants in this universe, and $\overline{C}'$ is the new set of constants being considered. The variation therefore consists of a magnitude, δC, and a direction $\delta\hat{C}$. The latter comprises N-1 degrees of freedom. A special case is when the direction of change is chosen parallel to one of the coordinate system axes. In other words, when one constant is varied and the other N-1 constants are held fixed. But this *is* a special case. In general, variations in any direction should be considered. After all, an alternative set of universal constants could have been chosen, and these would result in a different coordinate system spanning the same parameter space. We have already presented an example. Thus, holding G fixed and varying c leads to a change of gravity, but varying c whilst holding G’ fixed does not. They are simply changes in different directions in parameter space.

It will be seen that considering changes in more than one of the constants simultaneously is crucial to a proper understanding of the nature of fine tuning.

§3.4 The 26 Constants of the Standard Model of Particle Physics

The fundamental particles which appear in the standard model of particle physics are described in Appendix B. The complete set of 26 constants of the standard model is listed in Table 3.2. This more complete set of constants will not be used in this book, and are included only for completeness. There are different ways of presenting the 26 constants. The scheme that has been adopted in Table 3.2 is to maximise the number of constants which are particle masses. This has the advantage that these constants then need no explanation. As a result, the first 16 constants in Table 3.2 are dimensionful (15 being masses). An alternative, and more common, convention is to use a set of constants all but one of which is dimensionless. The dimensionful constant is generally chosen to be v, the vacuum expectation value of the Higgs field. The first 12 mass parameters would then be replaced by the so-called Yukawa coupling constants (denoted G_u, G_d, etc in Table 3.2). But v uniquely determines the low-energy strength of the weak nuclear force, i.e., the Fermi constant $G_F = \left(\sqrt{2} \cdot v^2\right)^{-1}$. Our set of constants uses G_F rather than v since G_F is more transparent, relating directly to measurable quantities. Similarly, the dimensionless weak coupling constant, g, and the Weinberg angle, θ_W, would be conventionally chosen, whereas Table 3.2 favours the masses of the W and Z bosons as being more readily understood.

The last nine constants in Table 3.2 are more difficult to interpret, being rather deeply buried in the quantum field theoretical formalism. They comprise two sets of four, plus one more. The first set of four constants describes how the quarks are ‘mixed’. The second set describes how the neutrinos are mixed. The final constant is the so-called “CP-violating QCD vacuum phase”. I need to swot up on these and give a brief description. ….put some words of explanation in here. Wasn't there a good Physics World article on this?

For practical purposes it is more convenient to employ the neutron and proton masses in the set of constants (as in our Table 3.1). In principle, these are derivable from the standard model parameters, but the required lattice QCD calculations are extremely computationally challenging. Impressive progress has been made in recent years in deriving hadron masses from the standard model [see Christine Davies (2006) and add other Refs]. This is very important in terms of confirming the fundamental soundness of the standard model, but it offers no advantage for our purposes.

Table 5.2 lists a rather small upper bound for the neutrino masses (<0.4eV). This is derived from WMAP data, augmented by other astronomical data, see Goobar et al (2006) and Spergel et al (2006). The Particle Data Group still list far bigger upper bounds for the neutrino masses (Reference). However, measurements of neutrino oscillations give even smaller values for the mass differences (strictly $\sqrt{m_{\nu 1}^2 - m_{\nu 2}^2}$). Whilst this is still compatible in principle with large, but nearly equal, neutrino masses, this does not seem likely. It seems most likely that the neutrinos have masses less than 0.07 eV [see Mohapatra et al (2005)].

Note that the electromagnetic fine structure constant (α), or equivalently the quantum of charge, e, do *not* feature in the list of 26 constants. This is because we have chosen to include G_F and the W and Z masses as fundamental constants, and, in electroweak theory, the charge e can be expressed in terms of the Fermi constant and the W and Z masses. Thus,

$$e = 2^{5/4} M_W \sqrt{G_F} \left[1 - \left(\frac{M_W}{M_Z} \right)^2 \right]^{1/2}$$

Table 3.3 shows how dependent parameters are derived from the set of 26 constnats in Table 3.2.

Most obviously, and lamentably, the standard model requires more constants rather than less. The 26 constants of Table 3.2 are represented in our set of constant, Table 3.1, by just 5 physical constants. Having said this, we may cheat a little and occasionally sneak the up and down quark masses, and the pion mass, into our discussions.

§3.5 Other Cosmological Constants

Table 3.1 contains only 4 parameters to define the large scale structure of the universe. Whilst these are undoubtedly the four most significant parameters, several more may be required to define the finer detail of cosmological observations. For example, Tegmack, Aguirre, Rees and Wiczek (2006) list 11 parameters. One is the dark energy 'equation of state', which measures the relative contribution of the dark energy density (w) and the dark energy pressure. Based on WMAP data [Spergel (2006)] w appears to be close to -1, the value expected if dark energy can be represented by a cosmological constant (see chapter ?). Another parameter measures the curvature of space. As we have already noted, this is currently consistent with space being flat (though this might mean that the radius of curvature is very large).

Another possibly parameter might be the contribution of neutrinos to the mean density of the universe. Table 3.1 contains the density parameter for combined cold dark matter and neutrinos, $\Omega_{d+\nu}$, but not a density for neutrinos alone. The number of thermal neutrinos originating from the Big Bang is known, but only so long as the number of neutrino flavours is known (currently believed to be 3). The other uncertainty regarding the neutrino contribution is whether the universe might have a non-zero lepton number. If so, there might be more neutrinos (or antineutrinos) than is implied by a black body distribution. So the neutrino density is another possible cosmological parameter. Finally, there are several more parameters than can be added to refine the description of the all-important anisotropies in the cosmic microwave background radiation. Having made these observations, there will be no further consideration of these refinements in this book.

Last Update: 5th January 2008

Table 3.1: Our 13 Low Energy Universal Constants

Those chosen to define the system of units shown blue. Numbers in brackets give 98% confidence error bars on last digit(s)

Constant	Value in MKS or MKSA	Value in Planck units	Value in MeV^x	Alternative Dimensionless Form
c	2.99792458 x 10^8 ms^{-1}	1	-	-
$\hbar$	1.0545717(±4) x 10^{-34} Js	1	-	-
G	6.674(±2) x 10^{-11} $m^3kg^{-1}s^{-2}$	1	-	$\alpha_G = \frac{GM_P^2}{\hbar c} = 5.906 \times 10^{-39}$
$\mathbf{G_F}$	1.43584(±3) x 10^{-62} Jm^3	$\frac{G_F M_{PL}^2 c}{\hbar^3} = 1.7386 \times 10^{33}$	1.16637(±2) x 10^{-5} GeV^{-2}	$\alpha_w^p = \frac{G_F M_p^2 c}{\hbar^3} = 1.02682 \times 10^{-5}$
e	$5.384384 \times 10^{-14} \sqrt{Jm}$ (1) 1.6021765(±3) x 10^{-19} Coulomb	$\hat{e} = \frac{e}{\sqrt{\hbar c}} = 0.30282212$	-	$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.035999}$
$\mathbf{g_s}$	13.5(2) (dimensionless)	-	-	$\alpha_s = \frac{g_s^2}{4\pi} = 14.4$ (2)
$\mathbf{M_n}$	1.6749265(±3) x 10^{-27} kg	7.6955 x 10^{-20}	939.56536(±16) MeV	$M_n / M_p = 1 + \Delta_n$; $\Delta_n = 0.001378$
M_p	1.6726217(±6) x 10^{-27} kg	7.6849 x 10^{-20}	938.27203(±16) MeV	-
$\mathbf{m_e}$	9.109380(±3) x 10^{-31} kg	4.1853 x 10^{-23}	0.5109989(±1) MeV	$\beta = m_e / M_p$ = 1/1836.15
ρ_b (4) ρ_γ (5)	0.25 m^{-3} ~4 x 10^8 m^{-3}	1.06 x 10^{-105} 2.0 x 10^{-96}	- -	ξ = 1.6(±0.1) x 10^9 $\mathbf{\Omega_b}$ = 0.042(±0.002)
ρ_{d+v} (3)	0.20 x 10^{-26} kgm^{-3}	0.39 x 10^{-123}	-	$\mathbf{\Omega_{d+v}}$ = 0.218(±0.024)
ρ_λ (6)	0.76 x 10^{-26} kgm^{-3}	1.47 x 10^{-123}	-	$\mathbf{\Omega_\lambda}$ = 0.74(±0.04)
Q	2 x 10^{-5} (dimensionless)	-	-	2(±0.2) x 10^{-5}

(1)This is the MKSA charge divided by $\sqrt{\varepsilon_0}$, where $\varepsilon_0 = 8.854187817 \times 10^{-12}$ Fm^{-1}. The values for e here relate to low energies. Its value increases at larger energies (see Section 5.6).

(2)This is the low energy value relevant to the inter-nucleon force. At energies equal to the Z boson mass (~91GeV) g_s reduces to 1.221 (α_s = 0.1186). g_s falls below 1 at energies above 3100 GeV.

(3)Density of dark matter and neurinos is *not* constant. Deduced using $\Omega_{d+\nu}$ and a critical density at the current epoch of 1.00 x 10^{-26} kgm^{-3} (equivalent to ~6 H atoms per m^3).
(4)Average number density of baryons (nucleons) in the present epoch. This corresponds to a mass density normalised by the critical density of $\Omega_b = 0.042$ (±0.0015). The sum of the baryon, dark matter and dark energy densities equals the critical density to within ~1%.
(5)Average number density of photons in the present epoch.
(6)Dark energy appears to have an equation of state of close to -1, implying that it is associated with a pressure which is equal and opposite to this density (times c^2). The dark energy density is equivalent to ~4.5 H-atoms per m^3.

Table 3.2: The 26 Universal Constants of the Standard Model of Particle Physics (c=1)

Constant	Description	Algebraic Equivalent	Value in MKS	Value in MeV^x
M_u	Up quark mass	$vG_u/\sqrt{2}$	2.7 to 7.1 x 10^{-30} kg	1.5 to 4 MeV
M_d	Down quark mass	$vG_d/\sqrt{2}$	7.1 to 14.2 x 10^{-30} kg	4 to 8 MeV
M_s	Strange quark mass	$vG_s/\sqrt{2}$	1.4 to 2.3 x 10^{-28} kg	80 to 130 MeV
M_c	Charm quark mass	$vG_c/\sqrt{2}$	2.0 to 2.4 x 10^{-27} kg	1.15 to 1.35 GeV
M_b	Bottom quark mass	$vG_b/\sqrt{2}$	7.3 to 7.8 x 10^{-27} kg	4.1 to 4.4 GeV (MS scheme)
M_t	Top quark mass	$vG_t/\sqrt{2}$	3.1(±1) x 10^{-25} kg	174 ± 5 GeV
m_e	Electron mass	$vG_e/\sqrt{2}$	9.109380(±3) x 10^{-31} kg	0.5109989(±1) MeV
M_μ	Muon mass	$vG_\mu/\sqrt{2}$	1.8835306(±2) x 10^{-28} kg	105. 65837(±1) MeV
M_τ	Tau mass	$vG_\tau/\sqrt{2}$	3.1678(±5) x 10^{-27} kg	1.7770(±3) GeV
$M_{\nu e}$	Electron-neutrino mass	$vG_{\nu_e}/\sqrt{2}$	<7 x 10^{-37} kg (see text)	<0.4 eV (see text)
$M_{\nu\mu}$	Muon-neutrino mass	$vG_{\nu_\mu}/\sqrt{2}$	<7 x 10^{-37} kg (see text)	<0.4 eV (see text)
$M_{\nu\tau}$	Tau-neutrino mass	$vG_{\nu_\tau}/\sqrt{2}$	<7 x 10^{-37} kg (see text)	<0.4 eV (see text)
m_H	Higgs boson mass	$\sqrt{-\mu^2/2}$	1.8 to 4.5 x 10^{-25} kg	Unknown: Perhaps 100-250 GeV
M_W	W boson mass	vg/2	1.4337(±13) x 10^{-25} kg	80.425(±76) GeV
M_Z	Z boson mass	$vg/2.\cos\Theta_w$	1.6255(±1) x 10^{-25} kg	91.188(±4) GeV
G_F	Fermi constant	$\left(\sqrt{2}\cdot v^2\right)^{-1}$	1.43584(±3) x 10^{-62} Jm^3	1.16637(±2) x 10^{-5} GeV^{-2}
g_s	Strong force coupling (dimensionless)	-	For energy < tens of MeV: g_s = 13.5 (α_s = 14.4)	
			For energy ~640 MeV: g_s = 3.545 (α_s = 1.0)	
			For energy = M_Z (~91 GeV): g_s = 1.221 (α_s = 0.1186)	
			For energy ~3100 GeV: g_s = 1.0 (α_s = 0.07958)	

Table 3.2 (continued): The 26 Universal Constants of the Standard Model of Particle Physics

Constant	Description	Value (dimensionless)
$\sin\theta_{12}$	Quark CKM matrix angle	0.2243(16)
$\sin\theta_{23}$	Quark CKM matrix angle	0.0413(15)
$\sin\theta_{13}$	Quark CKM matrix angle	0.0037(5)
δ_{13}	Quark CKM matrix phase	1.05(24)
$\sin\theta'_{12}$	Neutrino MNS matrix angle	0.55(6)
$\sin 2\theta'_{23}$	Neutrino MNS matrix angle	>0.94
$\sin\theta'_{13}$	Neutrino MNS matrix angle	<0.22
δ'_{13}	Neutrino MNS matrix phase	unknown
θ_{qcd}	CP-violating QCD vacuum phase	$< 10^{-9}$

Last Update: 5th January 2008

Table 3.3: Derived Constants from the Standard Model of Particle Physics

Constant	Description	Algebraic Equivalent	Value
$-\mu^2$	Quadratic Higgs potential coefficient	$2m_H^2$	Unknown: perhaps 2 x 10^4 to 1.2 x 10^5 GeV^2
v	Higgs vacuum expectation value	$\frac{1}{\sqrt{\sqrt{2}G_F}}$	246.7 GeV
λ	Quartic Higgs potential coefficient	$\frac{-\mu^2}{v^2}$	Unknown: perhaps ~1 (dimensionless)
g	Weak coupling constant	$\frac{2M_w}{v}$	At M_Z: 0.6518(±4) At 0 energy: 0.6425(±20)
α_w	Weak interaction strength	$\frac{g^2}{4\pi} = \frac{\sqrt{2}M_w^2}{\pi M_p^2}\alpha_w^p$	At M_Z: 0.03381(±5) At 0 energy: 0.03285(±22)
θ_w	Weinberg angle	$\cos^{-1}\left(\frac{M_w}{M_Z}\right)$	At M_Z: 0.5016(±4) At 0 energy: 0.4908(±20)
$\alpha = \frac{e^2}{4\pi\hbar c}$	Electromagnetic interaction strength (fine structure constant)	$\alpha_w \sin^2\theta_W$	At M_Z: 1/127.918(±36) At 0 energy: 1/137.035999(±3)
$\hat{e} = \frac{e}{\sqrt{\hbar c}}$	Electromagnetic coupling constant (dimensionless quantum of charge)	$\sqrt{4\pi\alpha}$	At M_Z: 0.313429(±44) At 0 energy: 0.30282212(±1)