CCC12B - Structure Formation: The Fine Tuned Primordial Fluctuations? Tegmark and Rees's Argument for the Fine Tuning of Q

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Within a hot Big Bang model, no structure would form unless some small non-uniformity were present from the start[1]. This initial small fluctuation in the density then acts as a seed, causing an escalating gravitational collapse. Or, at least, gravitational collapse will occur so long as other conditions are conducive.

The first of these conditions is that there must, as always, be a cooling mechanism. The second is that other dynamic factors which act in opposition to collapse are not dominant. There are potentially three such factors which oppose collapse. One is radiation. Whilst the radiation dominant era persists, structure cannot form due to the support provided against gravitational collapse by radiation pressure. The second factor is, in Friedmann equation parlance, the spatial curvature. This just means the expansion of universe, which gravity must overcome if structure is to form. And thirdly, there is the cosmological constant, Λ. Providing that Λ is positive (as it appears to be in our universe) it will cause an exponentially rapid expansion once it becomes the dominant term. Once the era of lambda dominance is entered, no structure can form thereafter.

However, the formation of structure is not anthropically sufficient. Assuming that we want to contrive conditions suitable for the evolution of planetary life, we also need stable planetary systems. The issue here is not so much the *formation* of planets, which can be assumed to occur, but the persistence over geological and evolutionary timescales of reasonably uniform physical conditions. A planet whose distance from its star varied randomly by a factor of, say, ten over timescales of millions, or hundreds of millions, of years, would repeatedly annihilate any evolving life forms due to the extreme changes in physical conditions. Such planetary disruption will be possible[2] if the system has a close encounter with another star. The implications of this are that close encounters between stars should occur no more often than roughly the lifetime of the stars. And, to permit evolution to occur, this is presumed to need to be of the order of billions of years.

Tegmark and Rees (1997) have examined the above factors in order to arrive at anthropic bounds on the parameter, Q (defined as the fractional fluctuation in the density). Their lower bound on Q is derived from examining the cooling mechanisms. If Bremsstrahlung were the relevant mechanism, requiring energies of the order of a Rydberg or greater, then a bound $Q \geq 10^{-8}$ can be derived. However, Tegmark & Rees claim that atomic 'line cooling' is the relevant mechanism. This involves free electrons exciting neutral hydrogen atoms from their ground state into their first excited state, followed by emission of a Lyman-α photon. Tegmark and Rees use solutions from the literature to find the cooling timescales by this mechanism as a function of temperature. Equating this with a

[1] Aguirre (2001) states that, in a cold Big Bang model, structure can form even with zero Q.

[2] Whether planetary disruption is inevitable in a close encounter, defined as an incoming star passing within the order of an astronomical unit, is another matter. Tegmark & Rees are cautious about this, since in some cases the disruption might be minor.

free-fall timescale gives an upper bound for how long this cooling mechanism can be effective (t_{cool}^{max}).

Now fluctuations grow proportionally to the size scale of the universe (starting at matter-radiation equality), and hence are proportional to $t^{2/3}$ in the matter dominated era. To grow from an initial density contrast of Q to a density contrast of order unity requires a factor of increase in the density contrast of 1/Q. Consequently, the time to virialise is given by $t^{2/3} \propto 1/Q$, that is $t \propto Q^{-3/2}$. This virialising time must be less than the maximum cooling timescale, t_{cool}^{max}. The consequence is a lower bound on Q which turns out to be $Q \geq 10^{-6}$. In this universe, Q exceeds this lower bound by just one order of magnitude.

It is worth noting that Tegmark & Rees dismiss molecular cooling as irrelevant to their argument. Molecular cooling is effective at much lower energies, and hence lower temperatures. It is worth noting that the cold Big Bang alternative universe of Aguirre (2001) is based on structure formation through such molecular cooling mechanisms. He claims that these mechanisms are efficient under the conditions of higher density and lower temperatures prevailing in his universe. Presumably this is what allows structure to form in the Aguirre universe even though Q is only 10^{-11} to 10^{-8}, in gross violation of the above bound.

Tegmark & Rees go on to discuss the implications of assuming Q >> 10^{-5}. They first observe that this would eliminate the usual characteristic mass scale for galaxies. A rationalisation of the typical size of galaxies (in this universe) can be made by assuming that cooling is dominated by Bremsstrahlung and appealing to the requirement that the cooling timescale be less than the free-fall timescale if fragmentation is to occur. Details have been given in Chapter ? "About the Size of Things" where it is shown that this argument (due to Carr and Rees (1979), and others) leads to galaxies consisting typically of ~10^{12} solar masses, as observed. However, Tegmark & Rees observe that, for $Q > 3 \times 10^{-5}$, the cosmic microwave background photons provide an efficient mechanism for cooling via Compton scattering. This mechanism applies to all potential galactic mass scales. However, since collapse cannot occur before matter-radiation equality (t_{eq}), the first generation of galaxies will have masses corresponding to the horizon mass at the epoch t_{eq}, i.e. about 10^{16} solar masses. It is curious to note how close our universe ($Q \sim 10^{-5}$) is to having galaxies which are 10,000 times larger, and possibly larger still.

Tegmark & Rees derive an estimate for the time interval between stellar close encounters. This depends, of course, on the stellar density and 'fly-by' velocity. For these, the horizon scale galaxies discussed above are assumed. They conclude that the 'disruption' time interval is proportional to $Q^{-7/2}$, and hence decreases rapidly as larger Q values are considered. By setting the disruption time equal to some minimum which is anthropically acceptable (taken as a billion years), they conclude that Q is required to be less than 10^{-4}. In this universe, Q is less than this upper bound by just one order of magnitude. [In our universe, the disruption time is several orders of magnitude longer than the current age of the universe. Note that there is a double benefit of having $Q < 3 \times 10^{-5}$, since galaxies are also far smaller].

Thus, Tegmark and Rees conclude that the primordial fluctuations must lie in the range $10^{-6} < Q < 10^{-4}$ to be consistent with the emergence of life. However, this assumes that all other cosmological parameters are unchanged. Tegmark and Rees go on to discuss the implications of also varying the curvature parameter (Ω) and the cosmological constant (Λ). As mentioned above, structure can form only whilst matter dominates both the curvature and the cosmological constant terms in the Friedmann equation. Since all these terms vary with time (i.e. with the expansion size scale) in different ways, matter only remains dominant for a finite period. The time (or size scale) at which curvature becomes dominant can be found, as can the time for dominance by the cosmological constant. The requirement that the virialisation time be less than these timescales provides two distinct anthropic constraints, one on each of the curvature and the cosmological constant. These can be used to obtain rather loose bounds on the "matter-to-photon" ratio, which in Tegmark & Rees's definition depends upon the curvature parameter, Ω, as well as the photon:baryon ratio. Although not expressed in this way by Tegmark & Rees, the implications for the photon:baryon ratio can be written:-

$$10^{-4} < \frac{\xi}{\xi_0} < 10^3 \qquad \text{or} \qquad 10^5 < \xi < 10^{12}$$

Finally, the upper limit on the cosmological constant consistent with structure formation (i.e. consistent with matter dominance over the virialisation timescale) is proportional to Q^3. Thus, increasing Λ by a factor of 10^6 can be offset by increasing Q by a factor of 100. In such a universe, galaxy formation would be complete in a few million years. Thereafter the universe would expand exponentially under the influence of the dominant Λ. Between then and now it would inflate by a factor of $\sim e^{100}$, with the consequence that our galaxy would be the only thing in the observable universe.

This final observation of Tegmark and Rees is of particular significance since it undermines, or substantially weakens, the arguments of Martel et al discussed in CCC12A. Martel et al (1998) derived the most likely value for the cosmological constant consistent with the anthropic constraint. However, they did so assuming all other cosmological parameters were invariant. Tegmark & Rees point out that this is invalid, since variations in Λ by some factor f can be neutralised by assuming a simultaneous variation of Q by a factor $f^{1/3}$. Whilst Tegmark & Rees's bounds on Q are relatively tight in the context of order-of-magnitude estimates, i.e. one order of magnitude either way, this still permits a variation by a factor of 1000 either way in Λ to be accommodated without anthropic disadvantage.

Rick’s Critique of the Cosmic Coincidences: CCC12B - Structure Formation: The Fine Tuned Primordial Fluctuations? Tegmark and Rees’s Argument for the Fine Tuning of Q