CCC12A – Structure Formation: The Fine Tuned Cosmological Constant? Martel, Shapiro and Weinberg's Anthropically Likely Values for Λ

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From the perspective of fundamental physics, the cosmological constant would be expected to be either of the order of the Planck density (i.e. enormous), or to be exactly zero due to some symmetry principle. The fact that astronomical evidence implies that the cosmological constant is small but non-zero is a puzzle. The fact that its magnitude happens to be comparable with the mean matter density of the universe in the current epoch is an additional puzzle.

From the anthropic point of view, a possible explanation runs as follows: Suppose there are a large number of universes, and that Λ takes a different random value in each one. The large majority of these universes would not give rise to life, since large values for Λ prevent structure formation. The reason is that a positive Λ causes gravitational repulsion. The formation of structure, i.e. galaxies and stars, requires gravitational collapse. But this would be frustrated by the effects of a positive Λ, unless it were sufficiently small. Hence, by our very existence, we have an observational bias towards abnormally small values of Λ. Specifically, under the condition that we exist, the only possible values for Λ are those which are less than some upper bound (Λ_{UB}) which is just consistent with structure formation. In the light of this observational bias, how surprised should we be at the value taken by Λ in our universe? The answer would seem to be that, so long as our Λ does not differ very much from Λ_{UB} in terms of order of magnitude (and providing, of course, that $\Lambda < \Lambda_{UB}$) then we should regard our Λ to be pretty much as we should expect.

Martel, Shapiro and Weinberg (1998) considered this issue of conditional probability in some detail. To do so it is necessary to take account of other factors influencing structure formation. The principal factor is the primordial density fluctuations which act as the seeds for gravitational collapse. Martel et al's approach is to assume that the magnitude of these density fluctuations is a given[1]. For a given level of fluctuation, and for any assumed Λ, they are then able to calculate, on the basis of a simple mechanical model, the fraction of the universe's matter which ends up condensed as galaxies / clusters. Their argument is that the probability of the universe giving rise to observers is proportional to this condensed mass fraction.

In this way, Martel et al can calculate the probability of Λ taking any stated value. In particular they derive the 5%'ile, 95%'ile and mean values of Λ. At the time of their original paper, the detailed data on fluctuations from WMAP was not available. Whilst COBE results were available, Martel et al concentrated mostly on predictions of the fluctuation amplitude from cold dark matter inflation models. Their result was that Ω_Λ would be expected to be greater than ~0.6. Perhaps more precisely expressed, their result

[1] This assumption has been criticised by Tegmark and Rees (1997), who point out that the anthropically allowed values for Λ are sensitive to the value assumed for Q. See CCC12B for details.

was that, if Ω_Λ should turn out to be less than 0.6, then this fact could not readily be explained by anthropic reasoning. In the event, WMAP measurements favour a value for Ω_Λ of ~0.7, broadly consistent with the anthropic argument.

More recently, Weinberg (2005) has updated the calculations using the fluctuation measurements from WMAP. He finds that the anthropic argument yields a probability that Ω_Λ ~ 0.7 of about 7% to 16%, depending upon the size scale used for the fluctuation smoothing (these results being based upon 1 to 2 Mpc respectively). No doubt the hoped-for answer was ~50%, but these probabilities seem large enough to maintain the credibility of anthropic reasoning.

In passing we note that Martel et al make no mention of the cooling mechanisms which permit the gravitational collapse to occur. I presume there is an unstated assumption that whatever cooling should be required will be forthcoming. It is not entirely clear to me that the demands on the cooling mechanism might not influence the outcome as regards the derived probability distribution for Λ. This is because the value taken by Λ will affect the rate of collapse, and hence will require differing constraints on the available cooling rate.

I also note that Martel et al have started with the assumption that Λ is positive. Whilst this appears to be true in our universe, it is not valid to assume it when calculating the probability distribution for Λ. Sufficiently large and negative Λ will be anthropically vetoed, since such a universe will not live long enough for life to evolve. However, one might expect structure formation to be enhanced by a sufficiently small but negative Λ. Provided that such universes have sufficiently long lifetimes they might contribute equally, or even dominate, the population of universes within the multiverse which give rise to life. In which case, Weinberg's estimate of 7% - 16% as the anthropic probability of our actual Ω_Λ (~0.7) may be substantially over-estimated. If so, this rebounds unfavourably on anthropic reasoning.

Finally, an issue which I do not personally understand as yet, and which I must investigate further, is the magnitude of Martel et al's σ, the standard deviation of the density fluctuations at the time of recombination. Their values for σ are, very crudely, of order 10^{-3}. But the usual value quoted for Q is around 10^{-5}. I don't understand the difference. Is it something to do with the reference time being considered? Or is it to do with the smoothing scale (Q, or the now-conventional σ_8, seem to be based on a scale of $8h^{-1}$ Mpc, which is much larger than Martel et al's smoothing scale. But would this make a difference of 2 orders of magnitude? I doubt it).