CCC 11 – The Photon:Baryon Ratio Coincidences – Recombination, Transparency and Freeze-Out

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1. Recombination

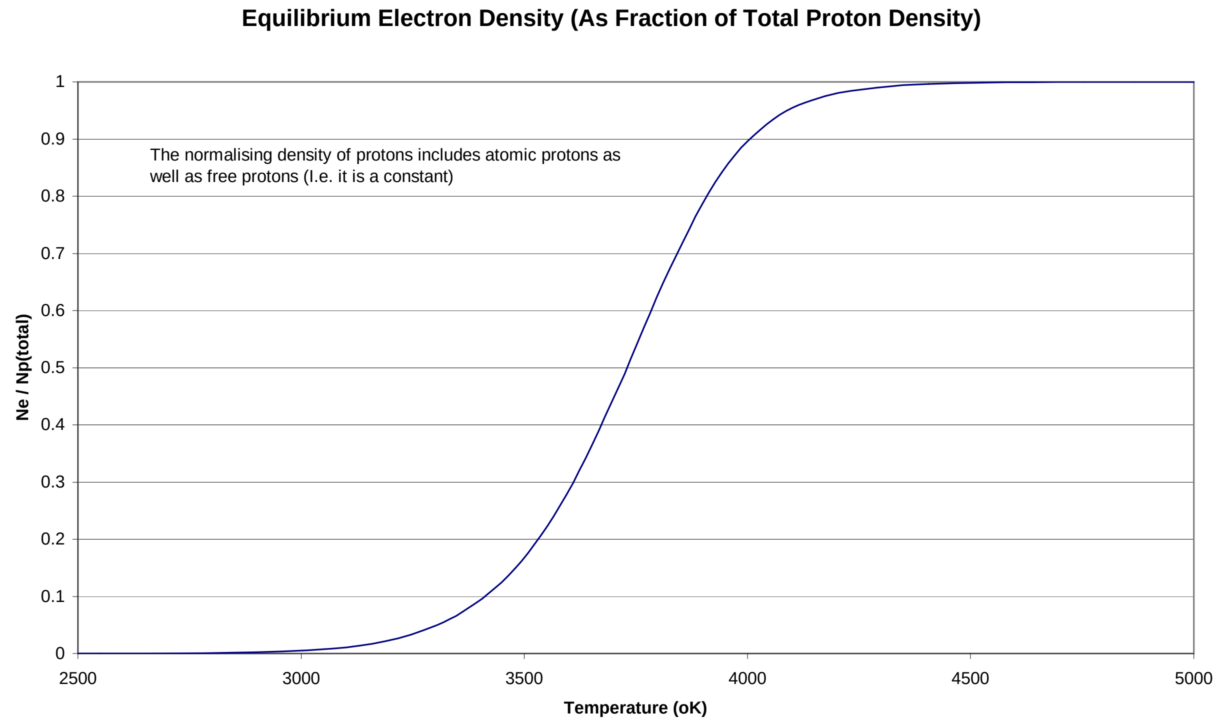
'Recombination' refers to the formation of neutral atoms when the nuclei created in the Big Bang capture electrons. The term 'recombination' is rather inappropriate since the nuclei and electrons have never previously been combined. The nuclei which are present, following Big Bang nucleosynthesis, are:-

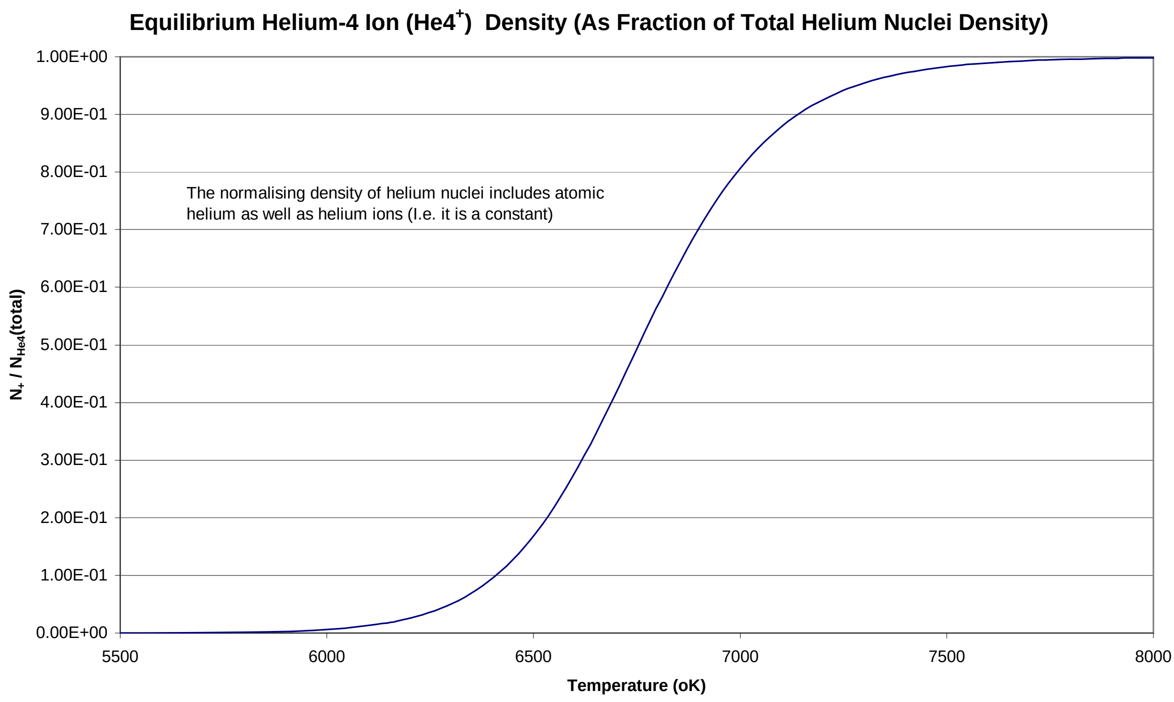
	Fraction by Mass	Fraction by Number
H	~75%	~92.3%
D	~0.014%	~0.009%
He4	~25%	~7.7%
He3	~0.003%	~0.001%
Li7	~10^{-7}%	~2 x 10^{-8}%

The propensity for these nuclei to capture electrons clearly depends upon their ionisation potentials (χ), i.e. the amount of energy released when a free electron is captured. If the typical photon energy (~$2.7k_BT$) exceeds the ionisation potential, the neutral atoms will not be stable but will immediately be re-ionised. Taking the ionisation potential of hydrogen for example (13.6eV), there will be no stable hydrogen atoms above a temperature of $13.6eV/2.7k_B \sim 58{,}000^oK$.

However, there continues to be essentially no neutral hydrogen even at temperatures of only one-tenth of this. The reason is mostly due to the huge numerical preponderance of photons over nuclei and electrons (i.e. the large photon:baryon ratio). The argument is familiar from Chapter 6 of the Tutorial. In that case we saw that deuterons became stable only when the number of photons with energies sufficient to cause deuterons to fission (E_f) became less than the number deuterons. Since the number of deuterons is only ~10^{-10} times the total number of photons, it requires only 1 in 10^{10} photons to have the necessary energy. This occurs at temperatures far lower than given by $2.7k_BT \sim E_f$.

The same approach is used in Chapter 8 of the Tutorial to obtain a temperature substantially lower than $\chi/2.7k_B$ at which neutral hydrogen, helium and lithium atoms are still unstable. However, the temperature at which neutral atoms are stable is lower still. This is because it is not necessary for a ground state electron to be ionised by a single photon. It may occur from multiple photon interactions. In Chapter 8 of the Tutorial, the equation which provides an accurate value for the concentrations of neutral and ionised species at any given temperature (known as the Saha equation) is derived from first principles. The resulting fractional concentration of free electrons is given by the graph which follows, as a function of temperature. Note that time runs from right to left on this graph. The fractional concentration of neutral hydrogen is just the complement of this (i.e. 1 – the fractional free electron concentration). The second graph shows the formation of neutral helium-4. This is essentially all over before hydrogen recombination starts, so the two processes do not compete. Also note that, whilst neutral lithium formation has yet to occur and hence there will be some excess free electrons which pair with these lithium nuclei, the amount of lithium is so small that this can safely be neglected.





The time-line of the formation of the various ions and neutral atoms can be summarised as follows:-

Atom or Ion	**1% Have Formed**		**99% Have Formed**	
	Temperature, ^oK	**Time, years**	**Temperature, ^oK**	**Time, years**
He_4^{+} *	-	-	19,000	9,000
Li_7^{+} *	-	-	17,000	11,000
Helium-4	7,620	57,000	6,030	91,000
Hydrogen	4,290	180,000	3,070	350,000
Lithium-7	1,055	3,000,000	850	4,500,000

*these cases refer to the temperatures/times at which the doubly charged ions combine to form the singly charged ions, and are very approximate (obtained from the estimate of Section 2 by subtracting 2,000^{o}K).

2. Transparency

Of course, there are degrees of transparency. The clearest glass will be opaque if thick enough. The standard of transparency we have in mind is an extreme one. Astronomers wish to be able to see the whole of the observable universe. This is not merely a tautology. In this context, the 'observable' universe is the region which can in principal be observed without violating the precepts of relativity. On the other hand, by 'see' we mean the detection of light, or other electromagnetic radiation, from the distant parts of the universe.

Hence, roughly speaking, the observable universe is of the order of the age of the universe times the speed of light. To be able to detect light originating from such a distance, the mean free path of photons must be comparable with, or larger than, the size of the observable universe. This is feasible only because the mean density of matter in the universe is so exceedingly small.

What interactions between the photons and matter can occur? Once all matter is in the form of neutral atoms in their ground state, the photons cannot interact with the bound electrons. This is because, to do so, the electrons would need to be raised to a higher energy level, there being no lower energy levels unoccupied (by definition of the ground state, and thanks to the exclusion principal). But we have seen in Chapter 8 of the Tutorial that, by this time, the photon energies are far too small to excite the atomic electrons. This, of course, is exactly why the neutral, ground state, atoms are stable.

Hence, when all matter is in the form of neutral atoms, the only possibility for photon interactions is with the atomic nuclei. We will see below that the cross sections for photon-nucleus interactions are very small compared with photon-electron interactions (that is, interactions with free electrons). Consequently, the time of recombination (~350,000 years, see above Table) is essentially the same as the time at which the universe becomes transparent. Actually, this is not obvious for the reasons we now discuss.

We have seen that by 350,000 years, virtually all helium nuclei and 99% of hydrogen nuclei have become neutral atoms. For this reason, 350,000 years is often taken as the time at which the universe became transparent. However, even after this time there will be some free protons and electrons left (less than 1 %). The fraction of remaining free protons (and electrons) falls to $<10^{-4}$ by 2574°K (462,000 years). However, even after that time, there are still lithium ions, and hence a matching number of free electrons, i.e. about 1 electron for every 10^8 initial free electrons, which persist until around 4 million years. In view of the persistent non-zero density of ions and free electrons, is the claim that the universe became transparent at ~350,000 years valid? In other words, is the *absolute* density of free electrons and ions sufficiently low after 350,000 years to render the universe transparent?

It will readily be seen that the question hinges on the size of the primordial photon:baryon ratio, since, for a given remnant fraction of free protons and electrons, this is what determines their absolute density.

To address this we need to estimate the mean free path of photons at different epochs, comparing it with the corresponding size of the observable universe. Equivalently, we need to estimate the typical time between photon interactions, and find when this first exceeds the age of the universe. Chapter 9 of the Tutorial has carried out this calculation, with the following result:-

time, t years	**Temperature, °K**	**Fraction of free electrons (y)**	**Interaction Time T_I, years**	**T_I / t**
10,000	18,200	1	113	0.006
30,000	10,500	1	580	0.019
100,000	5,750	1	3,570	0.036
180,000	4,290	0.99	8,650	0.048
220,000	3,880	0.75	15,500	0.070
238,000	3,730	0.50	26,000	0.11
260,000	3,570	0.25	60,000	0.23
286,000	3,400	0.10	173,000	0.60
300,000	3,320	0.057	325,000	1.08
314,000	3,246	0.033	600,000	1.92
350,000	3,070	0.010	2,350,000	6.7
*585,000**	*2,200*	*10^{-6} (1)*	*6×10^{10}*	*$\sim 10^5$*
*1,760,000**	*1,055*	*10^{-8} (1)*	*6×10^{13}*	*$\sim 2 \times 10^7$*
*2,440,000**	*850*	*10^{-10} (1)*	*$\sim 10^{16}$*	*$\sim 2 \times 10^9$*

*using $Tt^{2/3} = 1.54 \times 10^{12}$ for T<3000°K. Earlier times use $T\sqrt{t} = 1.02 \times 10^{10}$.

(1) These very low values are actually erroneous because freeze-out of hydrogen recombination occurs at $y \sim 7 \times 10^{-5}$ (see below), forming a lower limit for the concentration of free electrons.

The final column gives the interaction time as a fraction of the age of the universe. Hence, we see from the above Table that the universe does indeed become transparent over the same time period that hydrogen is recombining. The universe is opaque at the start of this period (say, 180,000 years) since the average photon will experience ~20 interactions during its passage across the universe. By the end of the period of hydrogen recombination, however (at, say, 350,000 years) most photons will circumnavigate the whole universe without interaction. The time at which half the photons interact in their circumnavigation is ~300,000 years. This is the nearest we can judge to the middle of the transition between opaque and transparent conditions (translucent?). Hence, the usual statement is confirmed.

However, it is the absolute density of free electrons that determines whether the universe is opaque or transparent. As well as depending upon the fraction of the original number of free electrons remaining, the absolute density of free electrons is proportional to the number primordial baryons, i.e. inversely proportional to the photon:baryon ratio – which has been assumed to be 1.9×10^9 in the above calculation. This raises the interesting questions,

(a) If the photon:baryon ratio were different, might the onset of universal transparency not coincide with recombination after all?;
(b) If the photon:baryon ratio were different, might the universe have remained opaque indefinitely – or, at least, for much longer?

The answer to both questions is "yes", as we shall see in the following Sections.

3. Recombination and Transparency Times' Dependence On Photon:Baryon Ratio

Firstly we re-calculate the recombination time for hydrogen for a range of different photon:baryon ratios, factoring the actual ratio by f, where $10^{-5} \le f \le 10^{+5}$. The temperatures at the onset of recombination (y = 0.99) and the near-completion of recombination (y = 0.01) are given in the Table below;

f	Temperature (^{o}K) for y = 0.99	Temperature (^{o}K) for y = 0.01
0.00001	6388	4029
0.0001	5815	3797
0.001	5337	3589
0.01	4933	3405
0.1	4585	3239
1	4290	3070
10	4030	2957
100	3791	2823
1000	3588	2713
10,000	3403	2609
100,000	3235	2508

These temperatures may be converted to times as usual using

$\sqrt{t(\text{secs})} = 1.021 \times 10^{10} / T$, or $Tt^{2/3} = 1.536 \times 10^{12}$ for $T < 3000^o$K.

We can now calculate the average photon-electron interaction time, Ti, in exactly the same way as in Section 2 except that the electron density in Equ.2 is factored down by f. We perform the calculation at the two temperatures calculated above, corresponding to the onset and end of recombination (y = 0.99 and y = 0.01 respectively). Thus, for each different photon:baryon ratio we can deduce whether the onset of universal transparency coincides with the period of hydrogen recombination (i.e. the times between y = 99% and y = 1%). In general it does not, as we see from the following Table,

p:b ratio factor f	**temp ^oK**	**y**	**Ti years**	**universe age, yrs**	**Ti / universe age**	
0.00001	6388	0.99	2.63E-02	81006	3.25E-07	opaque
0.00001	4029	0.01	1.04E+01	203634	5.10E-05	opaque
0.0001	5815	0.99	3.49E-01	97756	3.57E-06	opaque
0.0001	3797	0.01	1.24E+02	229279	5.41E-04	opaque
0.001	5337	0.99	4.51E+00	116051	3.89E-05	opaque
0.001	3589	0.01	1.47E+03	256624	5.72E-03	opaque
0.01	4933	0.99	5.71E+01	135838	4.20E-04	opaque
0.01	3405	0.01	1.72E+04	285109	6.03E-02	opaque
0.1	4585	0.99	7.11E+02	157241	4.52E-03	opaque
0.1	3239	0.01	2.00E+05	315082	6.34E-01	borderline
1	4290	0.99	8.68E+03	179610	4.83E-02	opaque
1	3070	0.01	2.35E+06	350726	6.69E+00	transparent
10	4030	0.99	1.05E+05	203533	5.15E-01	borderline
10	2957	0.01	2.62E+07	375000	6.98E+01	transparent
100	3791	0.99	1.26E+06	230005	5.47E+00	transparent
100	2823	0.01	3.02E+08	402500	7.50E+02	transparent
1000	3588	0.99	1.48E+07	256767	5.78E+01	transparent
1000	2713	0.01	3.40E+09	427000	7.96E+03	transparent
10000	3403	0.99	1.74E+08	285444	6.09E+02	transparent
10000	2609	0.01	3.82E+10	453000	8.40E+04	transparent
100000	3235	0.99	2.02E+09	315861	6.41E+03	transparent
100000	2508	0.01	4.30E+11	480600	8.90E+05	transparent

We see that changing the photon:baryon ratio by a factor of ten either way just preserves the coincidence between transparency and recombination, although the actual value (~1.9 x 10^9) makes the coincidence most accurate. However, changing the photon:baryon ratio by more than a factor of 10 either way would lead to different times (temperatures) for transparency and recombination. In short, the coincidence of transparency and recombination is an 'accident' which depends upon the particular (apparently contingent) value for the photon:baryon ratio adopted by our universe, ξ ~2 x 10^9.

4. The Freeze-Out of Hydrogen Recombination

In Section 1 the temperatures (times) at which the formation of neutral hydrogen atoms was complete up to some arbitrary percentage as calculated. The basis of these calculations was the assumption that the recombination reaction $e^- + p^+ \rightarrow H + N\gamma$ is in thermodynamic equilibrium with its reverse reaction. Hence, the concentration of free protons and electrons was found to reduce continuously. However, this does not take account of the possibility that the recombination reaction could freeze out due to cosmic expansion. In Chapter 9b of the Tutorial it is shown that freeze-out of the hydrogen recombination reaction does occur, though not until recombination is just over 99.99% complete. Consequently, the freeze-out makes no significant difference to the abundance of neutral atomic hydrogen at these times. Nevertheless, this freeze-out is extremely important. It determines the non-zero residual density of free electrons and protons. Whilst only around 1 in 10^4 of the initial electrons and protons remain free, this is believed to play an important role in the formation of the first stars.

The calculation proceeds by finding the time for a proton to capture an electron. This is then compared with the Hubble parameter as the criterion for freeze-out:-

Electron-Proton Capture Reaction Times T_{ep}

T (^{o}K)	**y**	**$1.5t_{universe}$ (years)** [(1)]	**T_{ep} (years)**
3070	0.0088	526,000	3,400
3000	0.005	551,000	6,400
2900	0.002	580,000	17,400
2800	0.00078	611,000	48,700
2700	0.00029	645,000	144,000
2600	9×10^{-5}	683,000	509,000
2586	8×10^{-5}	688,500	580,000
2574	7×10^{-5}	693,000	670,000
2562	6.1×10^{-5}	698,000	778,000
2500	3×10^{-5}	724,000	1,680,000
2400	8×10^{-6}	770,000	7×10^6
2320	2.6×10^{-6}	810,000	2×10^7
2200	$\sim 10^{-6}$	877,000	7×10^7

[(1)] Freeze-out of the capture reaction will occur if the reaction rate is less than the universe's expansion rate, H. But $t_{universe} = 2 / 3H$ in the matter dominated era, whereas $t_{universe} = 1 / 2H$ in the radiation era. This time marks the transition between the two. For sake of argument the reaction time, T_{ep}, is compared with $1/H = 1.5t_{universe.}$ It is clear from the above results that using $1/H = 2t$ as the freeze-out criterion would reduce the associated value of 'y' by less than a factor of 2. This Table has assumed a time-temperature relation $T\sqrt{t} = 1.02 \times 10^{10}$ k$\sqrt{s}$ at the earliest times, but giving way to the matter dominated expression $Tt^{2/3} = 1.536 \times 10^{12}$ for T<3000^{o}K.

The line in red indicates where freeze-out of the capture reaction occurs, at about a temperature of 2574^{o}K. Somewhat less than 1 in 10^4 of the initial electrons remain free at this point. This result implies that recombination of hydrogen actually ceases at this temperature, so the number of free electrons (and protons) at this temperature will survive for a much longer period, taking an active role later in star formation.

The absolute density of remnant free electrons and protons is $\sim 10^4$ per m^3 at this time. Of course, their density will continue to reduce as the universe expands, but the actual number of free electrons and protons will remain constant after this time. Constant, that is, until our assumption of small-scale homogeneity breaks down, when the first stars or galaxies or superclusters start to form by gravitational collapse.

5. Would The Universe Remain Opaque If $\xi_{\gamma N}$ Were Small Enough?

If the photon:baryon ratio were smaller, the absolute density of baryons, and hence also electrons, would be larger. Now the transparency, or otherwise, of the universe must depend upon the absolute density of free electrons. We have seen above that the universe becomes transparent at a time of ~300,000 years, a temperature of ~3320 K and a fraction of remnant free electrons of 0.057. The latter relates to an assumed $\xi_{\gamma N}$ of 1.9 x 10^9. Thus, if we changed $\xi_{\gamma N}$ to, say, 1.9 x 10^8, so there are ten times as many electrons in total, we would expect transparency to occur when the fraction of free electrons was 0.0057, roughly. Thus, we can reduce the required fraction of free electrons to the freeze-out fraction of 0.00007 by decreasing the photon:baryon ratio by about a factor of 1000 to about 1.9 x 10^6. This estimate can be checked by repeating the detailed calculations, as in the above Sections, for the reduced photon:baryon ratio. For photon:baryon ratios greater than about 1.9 x 10^6 the universe would therefore still be opaque when the abundance of free electrons reached its minimum level. Consequently, the universe would remain opaque forever.

6. Structure Formation

Why should we care if the universe is transparent or not? Clearly it would make astronomy impossible if the universe were opaque, but is there any more fundamental reason? The answer is emphatically, "yes". By "opaque" we actually mean that the ever present photons are still interacting with matter, specifically via the free electrons. But this means that the photons are imposing a radiation pressure on the matter constituent of the universe. (We are assuming that there is sufficient interaction between the free electrons and the neutral hydrogen and helium atoms that this radiation pressure is effectively communicated to them too, since they constitute the bulk of the ordinary matter). Now this radiation pressure is a dominant effect. It acts in opposition to the process of gravitational collapse, i.e. it provides a pressure support against collapse. Its dominance is such that, provided that it is operative at all, it will effectively prevent collapse and hence prevent structure formation. Thus, the true significance of the universe remaining "opaque" is that no stars or galaxies would form, and hence no life.

Of course, we have not demonstrated quantitatively here that the radiation pressure is sufficient to counter structure formation. This needs doing! Moreover, our criterion for transparency may be too stringent for this application. We have defined transparency by the mean free path of a photon being equal to the size of the universe. If we are concerned with structure formation over a size scale L, then perhaps we need only require that the photon mean free path exceeds L? For superclusters, this may be a factor of 1/400 of the size of the universe, and for individual galaxies a factor of ~1000 smaller still, and for individual stars, far, far smaller still. However, the matter is not clear.

7. Conclusions

Hydrogen recombination takes place mostly between 180,000 years and 350,000 years, when the temperature falls from 4,300 K to 3070 K and the free electron concentration falls from 99% to 1%.

Hydrogen recombination ceases at a temperature of 2574^oK (at 462,000 years) when the numbers of remnant free protons and electrons are a fraction 7 x 10^{-5} of their initial numbers.

Rick's Critique of the Cosmological Coincidences: CCC 11 – The Photon:Baryon Ratio Coincidences – Recombination, Transparency and Freeze-Out

Transparency of the universe occurs at ~300,000 years, when the temperature is ~3320 K, and the remnant fraction of free electrons is ~5.7%. Hence, transparency occurs during the period of most active hydrogen recombination.

If the photon:baryon ratio were reduced by more than a factor of ten, the universe would remain opaque during the period of most active hydrogen recombination (i.e. whilst 99% of the neutral hydrogen were being formed). If the photon:baryon ratio were increased by more than a factor of ten, the universe would already be transparent when the period of most active hydrogen recombination starts (i.e. before the neutral hydrogen level reaches 1%). Hence, there is a coincidence between the time of recombination and the time of transparency which depends upon the specific value for the photon:baryon ratio of this universe. It is not clear whether this is anthropically constrained, e.g. due to its implications for structure formation.

If the photon:baryon ratio were 1000 times smaller (i.e. 1000 times more baryons) then the universe would remain opaque forever. This is because the concentration of remnant free electrons when the recombination reaction freezes-out due to cosmic expansion would be large enough to prevent photons circumnavigating the universe uninterrupted.

Structure formation is likely to be prevented whilst the universe remains opaque due to the radiation pressure support which would prohibit gravitational collapse. Hence, there is an anthropic reason for the photon:baryon ratio being greater than 2×10^6 (i.e., less than $1/1000^{th}$ of its actual value).