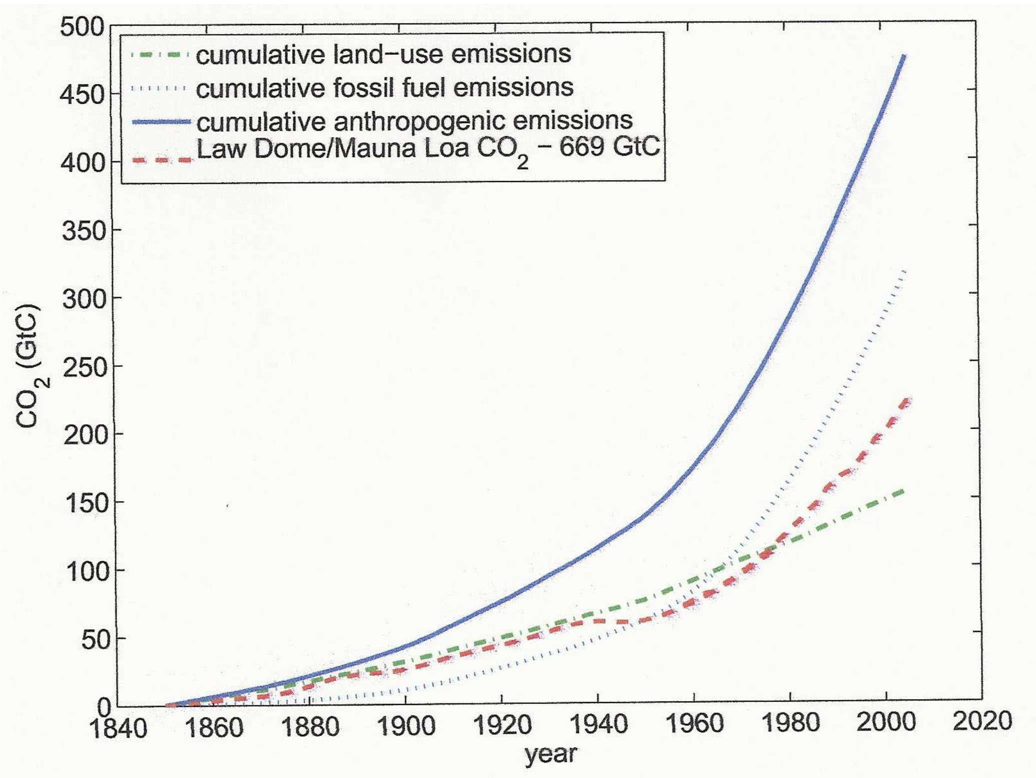
Atmospheric CO_2 Modelling: Is the IPCC Position Robust?
Rick Bradford, last update: 22/4/16

1. Introduction

That carbon dioxide levels in the atmosphere have been increasing is beyond doubt. CO_2 concentrations are easily measured, and accurate records exist going back a long way. However, how accurate are the projections of CO_2 concentrations in the future? And is it certain that increasing CO_2 levels are primarily due to the burning of fossil fuels? The proportionality between the curves of increasing atmospheric CO_2 concentration and rate of fossil fuel burning might seem sufficiently convincing in itself. But correlation is not causality, and mankind has had many other influences on the environment over the same period, not just the burning of fossil fuels. If you find the circumstantial evidence sufficiently compelling, it still only implies an anthropogenic origin for the atmospheric CO_2, not necessarily attributable to the burning of fossil fuels. The point is made by the following graph of increases in carbon since 1840. Human impact on land use might also have made a substantial contribution to net CO_2 emissions into the atmosphere.

Figure 1

Atmospheric carbon increase and anthropogenic carbon emissions since 1984. Taken from "On the Atmospheric Residence Time of Anthropogenically Sourced Carbon Dioxide" by Gavin C Cawley, University of East Anglia (in Energy & Fuels, October 2011)



It may seem perverse to look for alternative sources of atmospheric CO_2 increases when fossil fuel burning, aided by land use change, are perfectly sufficient without anything else. It seems less perverse when you recognise how huge are the natural fluxes of carbon. The 2014 IPCC report gives the following data (all relating to year 2011 and to carbon masses, not CO_2 masses),

(a) Flux into atmosphere from fossil fuel burning and cement production: 9.4 Gt/yr, plus a further ~1 Gt/yr due to changed land use;

(b) Flux into air from respiration and decay of vegetation and soil: 118.7 Gt/yr;

(c) Flux into land biota from atmosphere: 123 Gt/yr

(d) Flux into air from ocean: 78.4 Gt/yr;

(e) Flux into ocean from air: 80.0 Gt/yr.

Note how close to balance are (b) and (c). Similarly, note how close to balance are (d) and (e). This is not coincidence. It is because we are dealing here with a dynamical system which has evolved (human influences aside) to be in dynamic equilibrium - or, at least, close to equilibrium. The pre-industrial atmospheric CO_2 concentration, together with the ocean pH and land biota, have evolved to be in approximate steady state (ignoring seasonal variations and variations on geological timescales).

I draw your attention to the smallness of the anthropogenic flux (~10.4 Gt/yr) compared with the natural flux into the atmosphere (~197 Gt/yr). You see now why one might be sceptical about attributing atmospheric CO_2 increases to fossil fuel burning alone. A mere 5% change in the natural flux is equivalent to all fossil fuel burning, and it might reasonably be suggested that this could come about due to man's influence on the earth's flora. I presume this is what the ~1 Gt/yr "change in land usage" is supposed to represent - but in that case one can question its accuracy.

2. The Sceptics Position

There are those who doubt the anthropogenic origin of the increasing atmospheric CO_2, whether due to fossil fuels or otherwise. For example, the YouTuber "1000frolly" claims a non-anthropogenic origin. YouTubers alone could be dismissed out of hand, but there are credible, scientifically presented arguments also, for example Starr and Segalstad and Essenhigh, to name but three. It is worth spending some time examining the basis of their case, which is far from stupid (though, I shall ultimately conclude, incorrect). I initially thought their objections were valid, and it is instructive to understand why. My initial confusion was compounded because the attempt by Cawley to discredit the most recent sceptic, Essenhigh, was itself badly flawed.

All the sceptics focus strongly on the so-called residence time. This is the time it takes for a fraction $1/e = 37\%$ of the carbon in the atmosphere to be captured in the land and ocean sinks (or the reverse). It is the typical length of time a given CO_2 molecule stays in the atmosphere and is, as we shall see, about 5 years. The point of contention is embodied in the following quote from the 2104 IPCC report,

On an average, CO_2 molecules are exchanged between the atmosphere and the Earth surface every few years. This fast CO_2 cycling through the atmosphere is coupled to a slower cycling of carbon through land vegetation, litter and soils and the upper ocean (decades to centuries).

The climate change orthodoxy, as expressed by the above quote, is that, although the residence time of a given CO_2 molecule is indeed around 5 years, there is a second characteristic timescale - the typical residence time for anthropogenic additions of carbon - which is around 100 years or longer. The difference between these two timescales is very important because they indicate how long we must wait before the effects of discharges already made will ameliorate. The IPCC position is that, even if we stopped all fossil fuel burning completely today, we would have to wait more than a century to return to pre-industrial levels of CO_2 in the air. The sceptics position is that if empirical evidence favours a timescale of ~100 years or more then this indicates a non-anthropogenic origin for the excess CO_2, because the natural timescale is actually far smaller, i.e., ~5 years.

I initially sympathised with the sceptics because I too could not see why a dynamic system should display two different relaxation timescales. Why should an addition of anthropogenic CO_2 reside far longer than naturally occurring CO_2? It sounded like an outrageous fiddle to me. In fact there is a valid rationale. In part it has to do with the non-linear nature of the system in question. But in part it is also a confusion over what is meant by 'residence time'. At least, that is now my conclusion.

To illustrate the sceptics position, consider the following highly simplified model (which is not correct, please do note).

Suppose there are just two carbon sinks in the picture: the atmosphere and (say) the land based vegetation and soils. Let the total carbon in the atmosphere be C_a and the total carbon in vegetation-plus-soil be C_v. Suppose the rate at which photosynthesis or other mechanisms capture atmospheric carbon and transfer it to vegetation/soils is proportional to the carbon concentration in the air, i.e., proportional to C_a. Call this rate AC_a (Gt/yr), where A is some constant. Similarly, suppose the rate at which plant and soil respiration, and decay processes, return carbon to the atmosphere from vegetation/soil is proportional to the amount of such carbon-bearing matter, i.e.,, proportional to C_v. Call this rate BC_v (Gt/yr) where B is some constant.

To begin with let's assume this is a closed system with a fixed total amount of carbon, so that $C_{tot} = C_a + C_v$ is constant. Denoting rate quantities (time derivatives) by a dot, the dynamical equation is,

$$\dot{C}_v = -\dot{C}_a = AC_a - BC_v \tag{1}$$

Suppose at time $t = 0$ the amount of atmospheric carbon starts at some arbitrary level $C_a(0)$, and the amount of land-based carbon therefore starts at $C_v(0) = C_{tot} - C_a(0)$. The solution of the dynamical equation, (1), shows how these initial concentrations of carbon move towards the equilibrium concentrations,

$$C_v = \left(\frac{A}{A+B}\right)C_{tot} - \left[\left(\frac{A}{A+B}\right)C_{tot} - C_v(0)\right]\exp\{-(A+B)t\} \tag{2}$$

$$C_a = \left(\frac{B}{A+B}\right)C_{tot} - \left[\left(\frac{B}{A+B}\right)C_{tot} - C_a(0)\right]\exp\{-(A+B)t\} \tag{3}$$

We see that the time constant of this dynamic relaxation is,

$$\tau = \frac{1}{A+B} \tag{4}$$

For times long compared with τ the exponential terms in (2,3) become very small and the concentrations tend asymptotically to their equilibrium levels which we see are given by, at equilibrium: $$C_v = \left(\frac{A}{A+B}\right)C_{tot} \qquad C_a = \left(\frac{B}{A+B}\right)C_{tot} \tag{5}$$

Let's put some numerical values into these expressions. Using 2011 data, the total carbon in the atmosphere was 829 Gt. As given above, the flux of carbon into land biota from the atmosphere was about 123 Gt/yr at that time, giving $A = 123/829 = 0.148$ yr^{-1}.

The total amount of carbon in vegetation was ~550 Gt, plus a rather uncertain amount in soils, a central estimate being ~2000 Gt, making a total of perhaps 2550 Gt land-based carbon. The flux of carbon into the air from respiration and decay of vegetation and soil was 118.7 Gt/yr, and so we estimate $B = 118.7/2550 = 0.0465$ yr^{-1}.

Hence $A + B = 0.195$ yr^{-1} and so $\tau = 5.1$ years, in agreement with the usual estimate for the residence time of CO_2 in the atmosphere.

Moreover, using equs.(5), these data suggest equilibrium concentrations of $C_v = 2571$ and $C_a = 808$ Gt, which are very close to the actual values. Hence, the actual carbon concentrations and fluxes are very close to dynamic equilibrium. So far so good, this is what you would expect. This may beguile you into believing that the dynamic equation, (1), is a reasonable, if crude, model. But actually it is very misleading once out-of-equilibrium conditions are considered, as we shall see.

Using the model of equ.(1), what happens if we assume the system is not closed but instead there is, say, a steady rate of injection of new CO_2 from 'outside', i.e., an anthropogenic source? For simplicity consider a constant rate of increase of total CO_2, injected initially into the atmosphere. The total carbon is thus increasing linearly with time,

$$C_{tot} = C_{tot}(0) + \alpha t \tag{6}$$

In truth the rate of anthropogenic injection of carbon has been increasing steadily so that C_{tot} is closer to being a quadratic function of time - but that's a refinement we shall consider later. For now I wish only to establish the behaviour of the simple linear model of equ.(1). With additional CO_2 injected into the atmosphere at a steady rate α , the dynamical equations become,

$$\dot{C}_v = AC_a - BC_v \tag{7}$$

$$\dot{C}_a = -AC_a + BC_v + \alpha \tag{8}$$

Solving equs.(7,8) and now assuming that we start at time $t = 0$ from the equilibrium concentrations, we find the exact transient concentrations,

$$C_v = C_v(0) + \alpha A \tau^2 \left(\exp\left\{-\frac{t}{\tau}\right\} + \frac{t}{\tau} - 1 \right) \tag{9}$$

$$C_v = C_v(0) + \alpha t - \alpha A \tau^2 \left(\exp\left\{-\frac{t}{\tau}\right\} + \frac{t}{\tau} - 1 \right) \tag{10}$$

where τ is the same time constant as given by equ.(4). It is instructive to consider approximations to (9,10) in the short and long time limits. For $t << \tau$ we get,

$$t << \tau: \qquad C_v = C_v(0) + \frac{\alpha A}{2} t^2 \quad \text{and} \quad C_a = C_a(0) + \alpha t - \frac{\alpha A}{2} t^2 \qquad (11)$$

For example, the rate of injection of anthropogenic carbon (α) in 2011 was about 10.3 Gt/yr. Since $A = 0.148$/yr, if we had started from an equilibrium condition in 2011 the increase in carbon in the atmosphere in the first year (2012) would have been $10.3 - 10.3 \times 0.148/2 = 9.5$ Gt, whereas the extra carbon taken up by vegetation in that first year would be only $10.3 \times 0.148/2 = 0.8$ Gt. However, this is misleading because we actually start, in say 2011, from an already established dynamic condition. Since $\tau \approx 5$ years, we quickly enter the asymptotic region of equs.(9,10) in which the exponential terms vanish and we have,

$$t >> \tau \qquad C_v = C_v(0) + \alpha A \tau (t - \tau) \qquad (12)$$

$$t >> \tau \qquad C_a = C_a(0) + \alpha t - \alpha A \tau (t - \tau) \qquad (13)$$

Hence, for projections well beyond $\tau = 5$ years, the rate at which the carbon in the atmosphere increases drops from its initial rate of nearly α to only,

$$t >> \tau \qquad \dot{C}_a \to (1 - A\tau)\alpha = \frac{B}{A+B}\alpha \qquad (14)$$

which is only 24% of the rate of anthropogenic injection, hence only 2.5 Gt/yr. The balance of the injected carbon is being taken up by the land based sinks,

$$t >> \tau \qquad \dot{C}_v \to A\tau\alpha = \frac{A}{A+B}\alpha \qquad (15)$$

i.e., 76% of the anthropogenic carbon is being taken up harmlessly in land biota and soil.

Of course, these predictions from the model are at variance with the observed rate of increase of CO_2 in the atmosphere - measurements which are accurate and secure and indicate that about half the anthropogenic emissions remain in the atmosphere (and hence the latter is increasing at a rate of about 5 or 6 Gt/yr). The sceptics position is therefore that there must be a non-anthropogenic contribution to this atmospheric CO_2 which renders the above model inappropriate or incomplete.

Note how this conclusion depends crucially on the residence time, τ. Equs.(14,15) only become relevant when $t >> \tau$ - and we can claim they are relevant now only because our (overly simple) model has indicated that the residence time is so short, $\tau \approx 5$ years. If a very long residence time were to be justified, say around a century or more - as claimed by IPCC orthodoxy - then we would be in the $t << \tau$ regime in which case equ.(11) implies the rate of increase of atmospheric carbon would be a much larger proportion of the total anthropogenic injection rate, e.g., $\dot{C}_a = \alpha - \alpha A t$ = 85% after the first year.

This is why the argument over the effective residence time is key to the claims about the anthropogenic origin of atmospheric CO_2 increases. A very long residence time, of the order of 100 years or so, is crucial to the IPCC orthodoxy. And yet, our simple model appears to show that the residence time is really only ~5 years. Hence my initial view that the sceptics had a valid objection.

But it turns out that this is false. The matter hinges on the misleading simplicity of the model used here - specifically the assumption of linearity in the dependent variables.

3. An Improved Model

I will not make any great claims for the (still extremely simple) model which follows. Its sole purpose is to identify the serious error in the linear model of §2, and to show how the known rate of anthropogenic carbon injection can be seen to be consistent with the two secure pieces of empirical data: the measured increase in atmospheric CO_2 and the measured decrease in ocean pH.

The key is that the carbon fluxes do not vary linearly with the carbon concentrations. It simply is not true that vegetation will grow at twice the rate if the atmospheric CO_2 concentration is doubled. There are good biochemical reasons for this. One reason is that it is not necessarily the supply of CO_2 which throttles growth. Rather growth rates may be throttled by the availability of other nutrients, including reactive nitrogen and phosphorus - and, of course, water. But even ignoring those issues - which may be dominant - photosynthesis is not necessarily proportional to CO_2 concentration. A further limit will be sunlight, obviously. And yet another will be the size and density of leaf stomata which may also throttle the maximum reaction rate.

There has been a great deal of work investigating how growth rates depend upon CO_2 concentration, both from small scale controlled experiments and from field observations. Results vary, not surprisingly giving the large number of variables - often uncontrollable - and the wide range of plant species which may be studied. A guide to the effects is provided by the percentage increase in growth rate arising from a doubling of CO_2 concentration. Studies have indicated the following,

- Review of greenhouse studies by Poorter (Vegetatio 104/105: 77-97, 1993): 37%
- Greenhouse study by Wong (Oecologia, 1979, Vol 44, 68-74): 20% - 100%
- Outdoor experiments reviewed by Ainsworth and Long (New Phytol. 2005 Feb;165(2):351-71) and scaling to doubled CO_2 levels from experiments in the range 475-600 ppm: 27% - 62% (the scaling will be least reliable at the upper end).
- Franks et al (New Phytol. 2013, 197(4):1077-94) presents the graph reproduced below. Doubling CO_2 suggests a mean increased growth rate of perhaps around 30% - 40%, with considerable scatter.

Figure 2

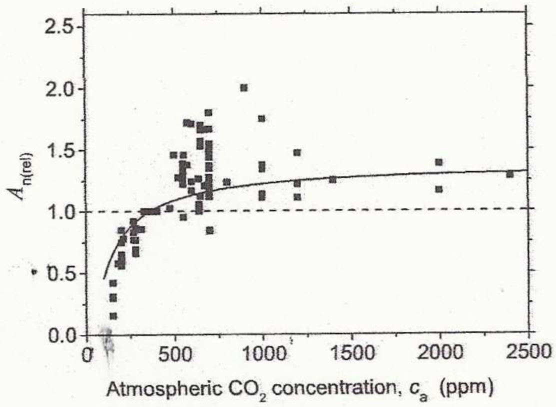


Fig. 4 Relative net CO_2 assimilation rate, $A_{n(rel)}$, vs the atmospheric CO_2 concentration in which plants were grown (all C_3). $A_{n(rel)}$ is calculated as the ratio of the assimilation rate in plants grown and measured at the growth CO_2 concentration, relative to that in plants grown and measured at ambient atmospheric CO_2 concentration. Data (black squares) are from numerous independent studies and surveys (see Table S1). The black line is the predicted relationship for long-term C_3 plant adaptation according to the hypothesis of optimal chloroplast nitrogen allocation under limiting nitrogen, using Eqn 5 with c_{a0} = 360 ppm and Γ^* = 40 ppm (see text for details).

In all cases the gains varied greatly with plant species, so there must be considerable uncertainty regarding the overall impact of raised CO_2 on terrestrial flora as a whole. The above graph suggests a power law dependence of growth rate on CO_2 concentration,

$$\text{growth rate, } f(C_a) = f_0\left(\frac{C_a}{C_a(0)}\right)^{\beta} \tag{16}$$

where the exponent β is 0.5 ± 0.25 corresponding to a doubling of CO_2 leading to growth increases in the range 20% to 67%, as implied by the above experiments.

The time datum will be taken as the pre-industrial era, when the atmospheric CO_2 concentration was 278 ppmv (or 589 Gt of carbon). The 2014 IPCC report indicates the total carbon take-up rate at this time was 108.9 Gt/yr, which sets the constant f_0. So the flux of CO_2 into land biota at any subsequent time is taken to be,

$$\text{growth rate } f(C_a) = 108.9\left(\frac{C_a}{589}\right)^{\beta} \quad \text{Gt/yr} \tag{17}$$

Similarly, we can anticipate that respiration and decay rates will not be simply proportional to the total land-based carbon sink, C_v, but will also be taken to vary according to some power law,

$$\text{respiration/decay rate } g(C_v) = g_0 C_v^{\alpha} \text{ Gt/yr} \tag{18}$$

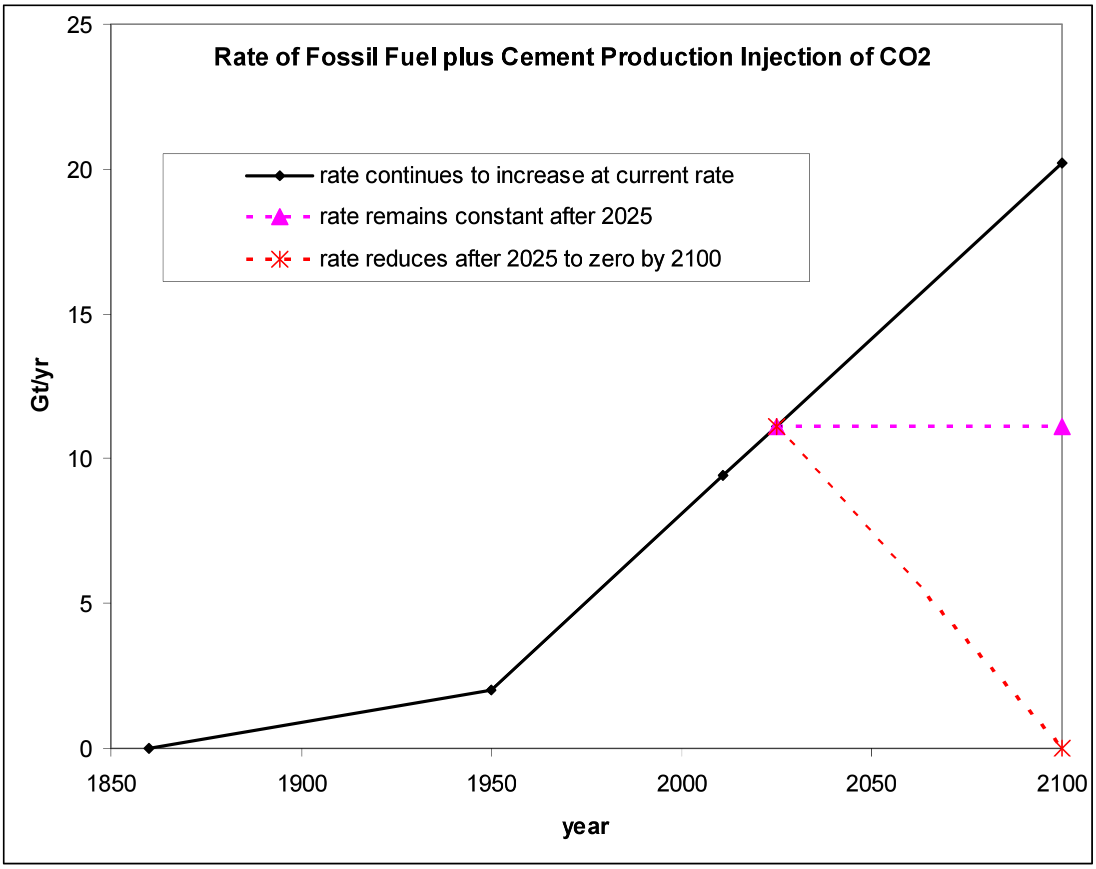
Different values of the exponent, α, will be explored in the model below. For any given exponent the coefficient, g_0, can be found from the inventory and flux of carbon at any time. For example, the 2011 data given above (2550 Gt of carbon and a respiration/decay flux of 118.7 Gt/yr) imply $g_0 = 2.35$ for an exponent of $\alpha = 0.5$.

In addition to the improved representation of the carbon fluxes into/out of the land based biota, the model to be used in this section will also make an allowance for the carbon uptake by the oceans. This is done on a very simple basis, assuming the orthodox (IPCC) net increase in ocean carbon at the present time of 1.6 Gt/yr. The reason why this is probably a secure figure, independent of modelling details, is that it is tied back to measured ocean pH decreases. For sake of argument I will assume this rate of net uptake has increased linearly with time since 1860, before which it was zero. I shall also assume a constant rate of 1.6 Gt/yr from 2011 onwards. Clearly this is rather crude, but it suffices for our present purposes, which is only to examine the credibility of the orthodox (IPCC) position. The net ocean rate of up-take is denoted $H(t)$. It is an explicit function of time.

Figure 3 gives the rate of injection of carbon into the air by fossil fuel burning plus cement production. To account for land use changes the total anthropogenic CO_2 injection will be taken to be 10% larger than the value read off Figure 3. The rate of anthropogenic CO_2 injection is assumed to increase at the current rate until 2025. Thereafter three different scenarios are modelled, as indicated in Figure 3.

The total anthropogenic injection rate is denote $F(t)$. It is an explicit function of time.

Figure 3



The dynamic equations are,

$$\dot{C}_a = F(t) + g(C_v) - f(C_a) - H(t) \qquad (19)$$

$$\dot{C}_v = -g(C_v) + f(C_a) \qquad (20)$$

4. Model Results

Time is started at the pre-industrial era, taken as 1860. The model parameters are constrained to produce results consistent with two secure empirical measurements: the current and historic atmospheric CO_2 concentrations, and the total take-up of carbon by the oceans as implied by ocean pH measurements. The latter is taken to be 129 Gt (consistent with the rate of net take-up as define in §3). The atmospheric CO_2 is as follows,

- In 1860: 589 Gt (278 ppmv)
- In 2011: 829 Gt (391 ppmv), or ~400 ppmv in 2015.

The first of these is the assumed starting condition. But the 2011 concentration of 391 ppmv is a result which the model is required to reproduce. This is done by tuning the exponent α in equ.18 as required.

Two values for the active land-based carbon are used: 550 Gt and 2550 Gt. The former assumes only vegetation takes part in respiration/decay whereas the latter assumes that soil plays an equal part. These data are the 2011 values, so iteration is required to find consistent 1860 data.

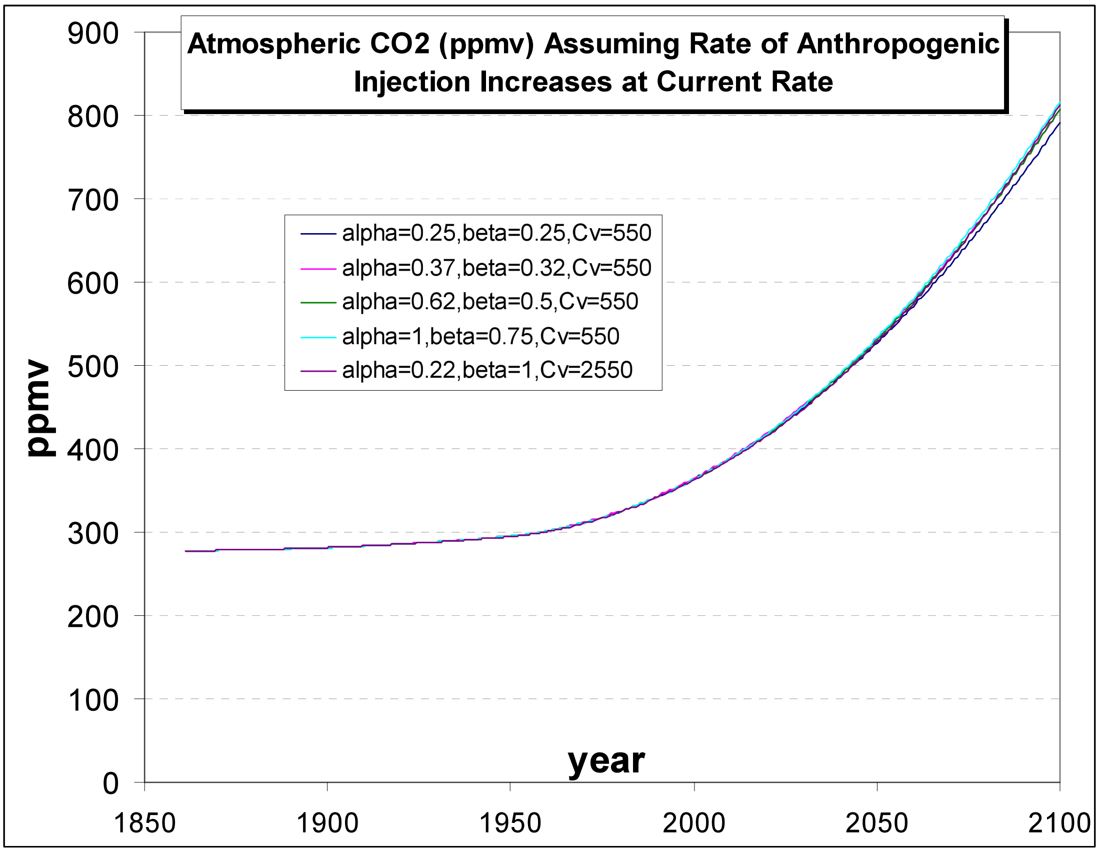
Table 1 gives the parameter combinations which were found to reproduce the current atmospheric CO_2 concentration. In all cases just over half the anthropogenic carbon resides in the atmosphere, with nearly one-quarter being transferred to each of the land and ocean sinks. This conclusion is in agreement with orthodox (IPCC) models. Using the larger estimate of land-based carbon is on the edge of possibility since the required β is at its upper bound (1) when the required α is at its lowest credible value (0.22). To reduce β would require an unrealistically small α.

Figure 4 gives the model predictions to the end of the century for all five of the possible cases defined by Table 1 (and assuming the worst case scenario that the rate of anthropogenic injection continues to increase at an unabated rate, reaching 22.2 Gt/yr, double its current rate, by the end of the century). Remarkably, all five model cases give virtually identical predictions. The requirement that the model correctly predicts the *current* CO_2 level appears to ensure that the model prediction for the end of the century is determined almost uniquely.

Table 1: Model parameters reproducing the current atmospheric CO_2

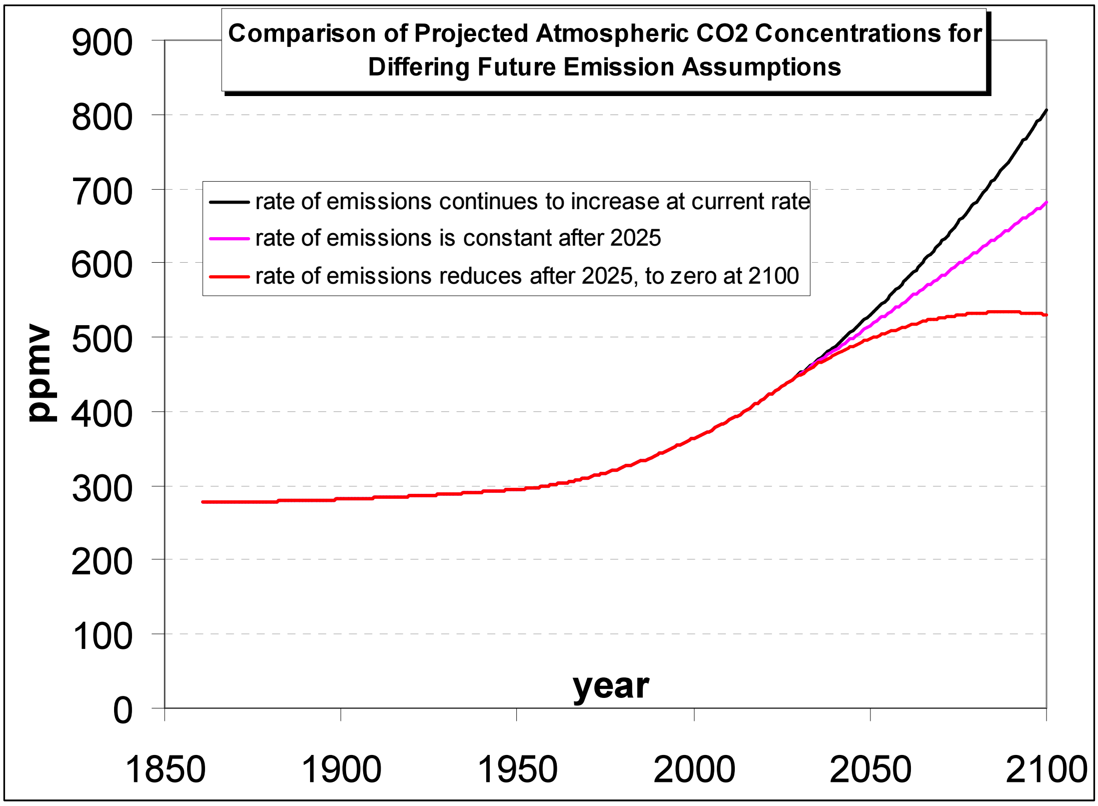
α	β	$C_v(1860)$	$C_v(2011)$
0.25	0.25	435	550
0.37	0.32	435	550
0.62	0.5	435	550
1	0.75	435	550
0.22	1	2430	2550

Figure 4


Atmospheric CO2 (ppmv) Assuming Rate of Anthropogenic Injection Increases at Current Rate
alpha=0.25,beta=0.25,Cv=550
alpha=0.37,beta=0.32,Cv=550
alpha=0.62,beta=0.5,Cv=550
alpha=1,beta=0.75,Cv=550
alpha=0.22,beta=1,Cv=2550
ppmv
year
0
100
200
300
400
500
600
700
800
900
1850
1900
1950
2000
2050
2100

The other scenarios for anthropogenic emission projections were explored using the central model with $\alpha = 0.62$, $\beta = 0.5$, $C_v(2011) = 550$ Gt. The result is shown in Figure 5.

Figure 5



The predictions of Figure 5 appear broadly similar to those of orthodox models. For example the various emission scenarios of Figure 6a are quoted here as giving the projected atmospheric concentrations of Figure 6b.

I conclude that my simple model confirms the reasonableness of the orthodox / IPCC models, and refutes the sceptics position. That my model predictions seem robust against variations in tunable parameters is particularly pleasing.

Figure 6a: from here

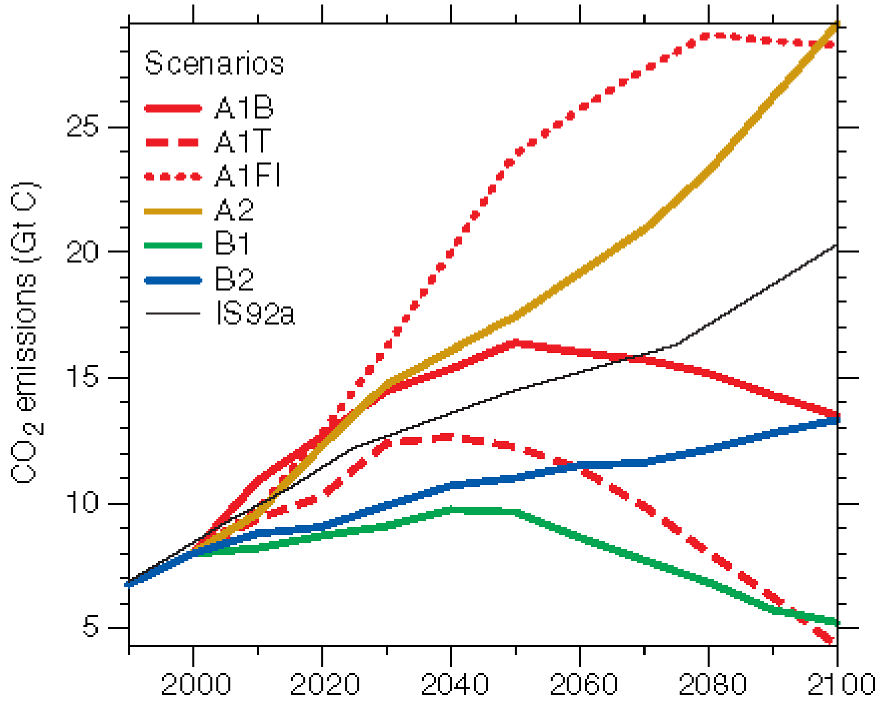
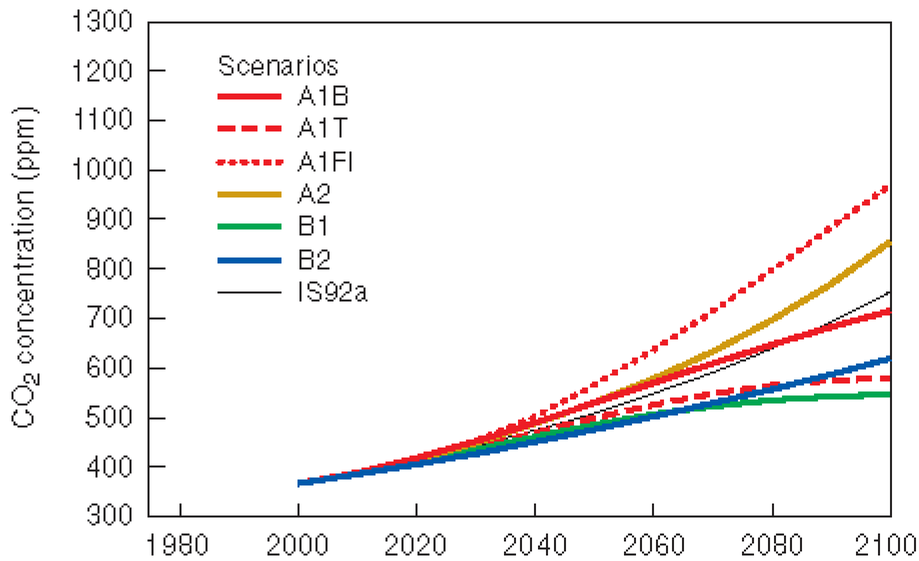


Figure 6b:
Atmospheric CO2 concentrations matching the emission scenarios of Fig.6a



5. What did the sceptics get wrong?

The loose end which needs to be tied up is: where did the sceptics' argument go wrong? I believe the answer lies in the non-linear dependence of the fluxes as expressed by equs.17 and 18. To see this, consider arbitrary fluxes out of, and into, the atmosphere: $f(C_a)$ and $g(C_v)$. Linearity may be restored, rendering the dynamic equations algebraically integrable, by retaining only the linear terms in a Taylor series expansion. Defining increments with respect to the equilibrium concentrations by,

$$C_a = C_a^E + \Delta C_a$$

$$C_v = C_v^E + \Delta C_v$$

where the equilibrium condition is $f\left(C_a^E\right) = g\left(C_v^E\right)$, allows the dynamic equation 20 to be approximated by,

$$\Delta\dot{C}_v = -\left(f'_E + g'_E\right)\Delta C_v + f'_E \Delta C_{tot} \tag{20}$$

where, $f'_E = \left.\frac{df}{dC_a}\right|_{C_a^E}$ and $g'_E = \left.\frac{dg}{dC_v}\right|_{C_v^E}$ and $\Delta C_{tot}(t)$ is the total anthropogenic emissions by time t, net of what has been absorbed in the oceans. For sake of illustration suppose we take a linearly increasing rate of net emissions, $F - H = rt$ so that $C_{tot} = rt^2/2$, then solving (20) assuming time $t = 0$ was the equilibrium condition with zero emissions, gives,

$$\Delta C_v = \frac{r}{2} f'_E \tilde{\tau} \left\{ t^2 - 2\tilde{\tau}t + 2\tilde{\tau}^2 \left(1 - \exp\left(-\frac{t}{\tilde{\tau}}\right)\right)\right\} \tag{21}$$

where the characteristic time is given by,

$$\tilde{\tau} = \frac{1}{f'_E + g'_E} \tag{22}$$

and, $$\Delta C_a = \Delta C_{tot} - \Delta C_v \tag{23}$$

For early times, $t << \tilde{\tau}$, we thus find,

$$t << \tilde{\tau} \qquad \Delta C_v \approx \frac{f'_E t}{3} \Delta C_{tot} \tag{24}$$

$$t << \tilde{\tau} \qquad \Delta C_a \approx \left(1 - \frac{f'_E t}{3}\right) \Delta C_{tot} \tag{25}$$

Assuming fluxes of the power law form, equs.(17,18), and using the case from Table 1 with $\alpha = \beta = 0.25$ we find $f'_E = 0.0328$, $g'_E = 0.054$, $\tilde{\tau} = 11.5$ years (using 2011 data). In common with our simple linear model, the residence time is still short, and we conclude that we are in the regime $t >> \tilde{\tau}$, for which the asymptotic expressions are,

$$t >> \tilde{\tau} \qquad \Delta C_v = f'_E \tilde{\tau} \Delta C_{tot} \tag{26}$$

$$t >> \tilde{\tau} \qquad \Delta C_a \approx \left(1 - f'_E \tilde{\tau}\right) \Delta C_{tot} \tag{27}$$

So the atmosphere is predicted in this regime to take up a fraction $\left(1 - f'_E \tilde{\tau}\right) =$ 62% of the anthropogenic emissions. This is a much better reproduction of the results of the numerical integration presented in §4.

Finally, then, what about the much longer characteristic time which is suggested by IPCC? I see this as a red herring. The residence time of a given CO_2 molecule is indeed short, the estimate of 5 years from the linear model only increasing to 11.5 years with the non-linear model. I believe the 'long' characteristic time is actually a completely different physical quantity. It refers physically to the time it would take the system to regain dynamic equilibrium if the anthropogenic emissions were switched off. The numerical model confirms that this is more than 100 years.

Another way of looking at this is that the molecular residence time is obtained, roughly, as the ratio $C_a / f(C_a)$, where f is the flux from atmosphere to land. But most of the carbon captured this way is simply re-emitted. If instead we are concerned with a net transfer of carbon from air to land, then the relevant timescale is more like $C_a /(f(C_a) - g(C\ )) = C_a / \xi F$, where F is the anthropogenic flux and ξ is the fraction of that flux which gets permanently fixed in land vegetation/soil. Hence this timescale is necessarily greater than $C_a / F \approx 829/10 = 83$ years, and probably several times longer still.

6. Conclusions

- I find against the sceptics and in favour of the orthodox (IPCC) position.
- My highly simplified model predicts that if the present rate of increase of anthropogenic CO_2 emissions continues indefinitely, then atmospheric CO_2 levels by 2100 will be about 800 ppmv, double current levels and nearly three times the pre-industrial level.
- If anthropogenic emission rates start to decline after 2025, reducing to zero by 2100, atmospheric CO_2 will never exceed 530 ppmv (33% higher than current levels, and just under double pre-industrial levels).