Are IPCC Models Stable Given Their Inclusion of Positive Feedbacks?

Rick Bradford, 13/9/19

The answer is yes.

Global temperatures vary a great deal spatially, diurnally and annually, and even the global average temperature is subject to some random variation. The climate is clearly stable against such fluctuations (by which I mean stable over short periods such as years, notwithstanding long term variations over centuries or longer).

Dynamical systems are stable because they have negative feedbacks, i.e., feedbacks which, upon a given fluctuation, react by negating the fluctuation and returning the system back towards its equilibrium.

In contrast, positive feedbacks cause a fluctuation to grow bigger and hence systems with positive feedback are unstable.

But IPCC climate models include positive feedbacks, most importantly those due to water vapour and clouds, which dominate the predicted global average temperature increases. These feedbacks are positive because, given an increase in temperature (due, in this case, to CO_2 radiative forcing) the resulting changes in water vapour and clouds cause (IPCC claim) the temperature to increase further. Specifically doubling CO_2 is predicted by the average of many models to cause 1.2°C increase in global average temperature (GAT) directly, plus a further 2.0°C increase due to these positive feedbacks (3.2°C in all).

This raises a concern that such models might be intrinsically unstable – and hence clearly incorrect.

That one cannot conclude immediately that the models are unstable is because there is another feedback effect in operation which is not usually referred to as a feedback in IPCC terminology but which crucially functions as such. This is simply the net power radiated from Earth into space. If temperature increases, more power will be radiated – an energy loss mechanism which will tend to reduce temperatures again, and hence is clearly a negative feedback.

In the simple toy model below I quantify the feedback effects and demonstrate that IPCC models are stable. The model is implicitly based on cloud feedback (only), and this is implemented via its effect upon albedo, a.

Let T be the temperature of the layer in the atmosphere at unit optical depth (approx. altitude 5km) which acts with unit emissivity.

The outgoing radiative power is $4\pi r^2 \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

The incoming power absorbed is $\pi r^2(1-a)S_0$, where $S_0 = 1367\ Wm^{-2}$.

I will assume that because of changes to cloud cover, or cloud properties (or indeed anything else) that the albedo is a function of temperature, $a(T)$.

At the equilibrium temperature, T_e, the incoming and outgoing radiated powers are in balance and hence $4\sigma T_e^4 = \big(1 - a(T_e)\big)S_0$ and if we assume that $a(T_e) = 0.3$ then this gives $T_e = 254.86K$.

If there is a temperature fluctuation to $T = T_e + \Delta T$ the incoming and outgoing powers will no longer be in balance. The net incoming power can be equated with the resulting rate-of-change of temperature times some appropriate heat capacity, C. The temperature varies throughout the atmosphere, of course, but one can imagine the "effective C" being a fudge for this. The resulting dynamic equation is thus,

$$C\frac{dT}{dt} = \pi r^2\left[\left(1 - a(T)\right)S_0 - 4\sigma T^4\right] \tag{1}$$

A short time τ after the fluctuation at time $t = 0$ the temperature will be given by,

$$T_e + \Delta T + \left.\frac{dT}{dt}\right|_{t=0}\tau \tag{2}$$

So the deviation from the initial equilibrium temperature after time τ has become,

$$\Delta T_1 = \Delta T + \left.\frac{dT}{dt}\right|_{t=0}\tau \tag{3}$$

Using (1) this can be written,

$$\Delta T_1 = \Delta T + \frac{\pi r^2}{C}\left[\left(1 - a(T_e + \Delta T)\right)S_0 - 4\sigma(T_e + \Delta T)^4\right]\tau \tag{4}$$

Retaining only first order terms in the small fluctuation ΔT inside the [ ] we approximate,

$$\left(1 - a(T_e + \Delta T)\right)S_0 - 4\sigma(T_e + \Delta T)^4 \approx \left(1 - a(T_e) - \left.\frac{da}{dT}\right|_{T_e}\Delta T\right)S_0 - 4\sigma(T_e^4 + 4T_e^3\Delta T)$$

Using the equilibrium condition that $4\sigma T_e^4 = \left(1 - a(T_e)\right)S_0$ this simplifies to,

$$\left(1 - a(T_e + \Delta T)\right)S_0 - 4\sigma(T_e + \Delta T)^4 \approx -\left[\left.\frac{da}{dT}\right|_{T_e}S_0 + 16\sigma T_e^3\right]\Delta T \tag{5}$$

Hence (4) becomes,

$$\Delta T_1 = \left\{1 - \frac{\pi r^2}{C}\tau\left[\left.\frac{da}{dT}\right|_{T_e}S_0 + 16\sigma T_e^3\right]\right\}\Delta T \tag{6}$$

In the same way we find that the temperature deviation from the equilibrium temperature after another time increment, τ, is,

$$\Delta T_2 = \left\{1 - \frac{\pi r^2}{C}\tau\left[\left.\frac{da}{dT}\right|_{T_e}S_0 + 16\sigma T_e^3\right]\right\}\Delta T_1 \tag{7}$$

And so on, so that after n time intervals of τ the deviation has become,

$$\Delta T_n = \left\{1 - \frac{\pi r^2}{C}\tau\left[\left.\frac{da}{dT}\right|_{T_e}S_0 + 16\sigma T_e^3\right]\right\}^n\Delta T \tag{8}$$

If the term in { } is less than one, then after a very large number of small time intervals, as $n \to \infty$, we see that $\Delta T_n \to 0$ and the initial equilibrium temperature is regained, i.e., the system is stable and relaxes back to its equilibrium after the fluctuation. On the other hand, if the term in { } is greater than one, then the deviation from the equilibrium becomes ever greater after each time interval and the system is unstable.

Thus, the system is stable if and only if,

$$\left.\frac{da}{dT}\right|_{T_e} S_0 + 16\sigma T_e^3 > 0 \tag{9}$$

Slightly confusingly, if the LHS of (9) is positive we get negative feedback, and hence stability, because of the minus sign in equ.(8).

If the albedo were found to increase as temperature increases (e.g., because cloud cover increases) then $\left.\frac{da}{dT}\right|_{T_e} > 0$ and condition (9) is clearly obeyed and the system is stable. This is as one would expect because increasing cloud cover is cooling and hence means a negative feedback (and this is the opposite of what IPCC assume).

However, even if $\left.\frac{da}{dT}\right|_{T_e} < 0$, perhaps because cloud cover decreases when temperatures increase – a positive feedback effect as IPCC assume – then the system may still be stable if the second term in (9) is larger in magnitude. Rewriting (9) and using $4\sigma T_e^4 = \big(1 - a(T_e)\big)S_0$ the stability condition becomes,

$$\left.\frac{da}{dT}\right|_{T_e} > -\frac{16\sigma T_e^3}{S_0} = -\frac{4\big(1-a(T_e)\big)}{T_e} = -\frac{4(1-0.3)}{254.86} = -0.0110\ K^{-1} \tag{10}$$

Thus, a model with a positive cloud (albedo) feedback effect, i.e., a negative $\left.\frac{da}{dT}\right|_{T_e}$, is still stable so long as the condition (10) is obeyed.

Using (10) we can show that the IPCC models are stable. Ground level temperature, T_g, is found by using an emissivity of $\varepsilon = 0.61$ in,

$$4\varepsilon\sigma T_g^4 = \big(1 - a(T_e)\big)S_0 \tag{11}$$

This gives $T_g = 288.38K$. Now IPCC claim 2.0K results from feedbacks (which we will attribute here entirely to albedo), giving $T_g = 290.38K$ which, using (11) again, implies a reduced albedo of 0.28, i.e., a change in albedo of $\Delta a = -0.02$. But this results from a total temperature change (say IPCC) of 3.2K, and so corresponds to a derivative $\left.\frac{da}{dT}\right|_{T_e}$ of $\frac{-0.02}{3.2} = -0.0063\ K^{-1}$ which respects the inequality, (10), and hence implies a stable dynamic.

Thus the IPCC models meet this simple stability criterion.

However, that does not mean that IPCC's treatment of feedbacks is necessarily correct, nor that the cloud or water vapour feedbacks are necessarily positive. It means only that one cannot rule the IPCC models out based on a failure to meet the necessary criterion of stability.

My position remains that the uncertainty over the magnitude of the cloud and water vapour feedbacks is very great, so that even their sign is uncertain. And, as a consequence, the evidence that the observed global warming is due to CO_2 is far weaker than the public are being led to believe.