Appendix A6 – Derivation of the Deuteron Photodisintegration Reaction Rate at a Given Temperature T

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The deuteron photodisintegration rate at a given temperature is required in Big Bang nucleosynthesis calculations. The cross section for $D + \gamma \to p + n$ at a given photon energy is calculable from Schrodinger wavefunctions (see Appendix A2). At a combined nucleon energy, W, which is small compared with the deuteron binding energy, B, the magnetic and electric dipole cross sections become,

$$\sigma_{dis}^{M} = \frac{2\pi\alpha}{3} L^2 \eta_M \sqrt{\frac{W}{B}} \qquad \sigma_{dis}^{E} = \frac{8\pi\alpha}{3} \rho_3^2 \eta_E \left(\frac{W}{B}\right)^{3/2} \tag{A6.1}$$

We are interested in temperatures below 10^9 K, which is sufficient to cover the whole of the BBN period. The average photon energy at this temperature, 2.7kT, is 0.233 MeV. This is small compared with the deuteron binding energy, B = 2.224 MeV. Only photons with energies greater than B can cause photodisintegration. Consequently, only a very small fraction of photons contribute to the reaction. The evaluation of the reaction rate therefore comes down mainly to an integral over the tail of the Planck distribution of photon energies.

Normalising the Planck spectrum by the total number of photons, of all energies, the fraction of photons with energies in the range E to E + dE is given by,

$$df_\gamma = 0.416 \frac{\varepsilon^2 d\varepsilon}{e^{\varepsilon} - 1} \qquad \text{where, } \varepsilon = \frac{E}{kT} \tag{A6.2}$$

and E is the photon energy. The (combined) nucleon energy is clearly given in terms of E by,

$$W = E - B \tag{A6.3}$$

For W << B, we can make the simplifying approximations,

- The electric dipole cross section is negligible compared with the magnetic cross section;
- The term ηM can be approximated as a constant 41.9, i.e. we use its limiting value for zero nucleon wavenumber, k.

The length scale, L, is given in terms of the proton and nucleon magnetic moments and equals 0.99 fm. Hence, the photodisintegration cross section for a photon of energy E is,

$$\sigma_{dis} \approx 0.63 \sqrt{\frac{E - B}{B}} \text{ fm}^2 \tag{A6.4}$$

This is converted to the reaction rate (per second per deuteron) by multiplying by the number density of photons times c. For photons in the energy range E to E + dE this factor is $\rho_\gamma c df_\gamma$, where df_γ is given by (A6.2) and ρ_γ is the total photon density, i.e. $\rho_\gamma = 0.2436\left(\frac{kT}{\hbar c}\right)^3$. Hence, the reaction rate, integrated over all photon energies, is,

$$R[T] \approx 0.2436c\left(\frac{kT}{\hbar c}\right)^3 \int_{\varepsilon=\varepsilon_{th}}^{\infty} 0.416\frac{\varepsilon^2 d\varepsilon}{e^\varepsilon - 1} 0.63x10^{-30}\sqrt{\frac{E-B}{B}} \quad (A6.5)$$

where we have converted the length dimensions to metres. The lower limit of the integration is $\varepsilon_{th} = B/kT$. Shifting the origin of integration to zero gives,

$$R[T] \approx \frac{1.92x10^{-23}}{\sqrt{\varepsilon_{th}}}\left(\frac{kT}{\hbar c}\right)^3 e^{-\varepsilon_{th}} \int_{x=0}^{\infty} e^{-x}\sqrt{x}\left(x+\varepsilon_{th}\right)^2 dx \quad (A6.6)$$

where the factor $kT/\hbar c$ is understood to be in m^{-3}. In deriving (A6.6) we have made use of the fact that $\varepsilon_{th} >> 1$ to simplify the exponential term. The integral in (A6.6) is easily evaluated,

$$\int_{x=0}^{\infty} e^{-x}\sqrt{x}\left(x+\varepsilon_{th}\right)^2 dx = \sqrt{\pi}\left\{\frac{15}{8}+\frac{3}{2}\varepsilon_{th}+\frac{1}{2}\varepsilon_{th}^2\right\} \quad (A6.7)$$

Hence, the deuteron photodisintegration rate at temperature T is,

$$R[T] \approx \frac{3.40x10^{-23}}{\sqrt{\varepsilon_{th}}}\left\{\frac{15}{8}+\frac{3}{2}\varepsilon_{th}+\frac{1}{2}\varepsilon_{th}^2\right\}\left(\frac{kT}{\hbar c}\right)^3 \exp\left\{-\varepsilon_{th}\right\} \quad (A6.8)$$

where $\varepsilon_{th} = B/kT$. Evaluation of (A6.8) over the range of temperatures relevant for BBN gives,

T (10^8 K)	**R[T] (s^{-1}) Equ.(A6.8)**	**R[T] (s^{-1}) NACRE web site**
4	4.87 x 10^{-15}	4.83 x 10^{-15}
5	2.70 x 10^{-9}	2.72 x 10^{-9}
7	1.17 x 10^{-2}	1.15 x 10^{-2}
10	1.31 x 10^{+3}	1.25 x 10^{+3}

The agreement with the NACRE data is excellent.