Analysis of AV Voting System

Rick Bradford, 24/4/11

In the 2010 UK General Election, the percentage of votes for the three principal parties were in the proportion 41% (Con), 33% (Lab), 26% (Lib), ignoring the smaller parties and rescaling to 100%. These figures contrast sharply with the proportion of seats won: 49% (Con), 42% (Lab), 9% (Lib). The proportion of seats won by the third party (Liberals) is far smaller than their proportion of the votes.

Simulations have been carried out based on the simplification that only the three major parties are involved, and based on random voting. It was assumed for simplicity that all voters placed all three candidates (though this is not a requirement of the proposed UK system). Three things are presented,

[1] Results assuming all three parties attract one-third of the total vote;

[2] Results assuming a weaker third party (representative of the UK);

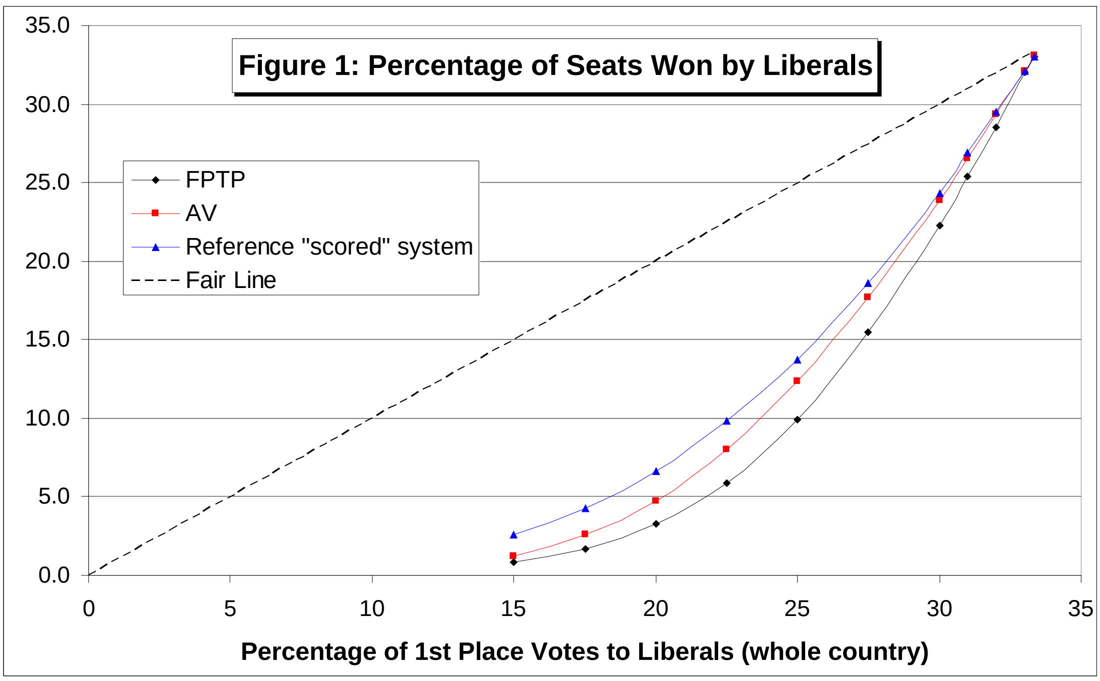
[3] Some known rigorous voting theorems.

1. All Parties Have One-Third of the Total Vote

Remarkably the AV and first-past-the-post (FPTP) system were found to produce a different winning candidate in 23% of cases. If this were characteristic of the UK (it isn't) it would suggest that the adoption of AV would have a dramatic effect on the resulting government.

2. Unequal Split of Votes Representative of the UK

Figure 1 plots the proportion of seats won by the 3rd Party (Liberals) against the proportion of 1st place votes they receive over the whole country.

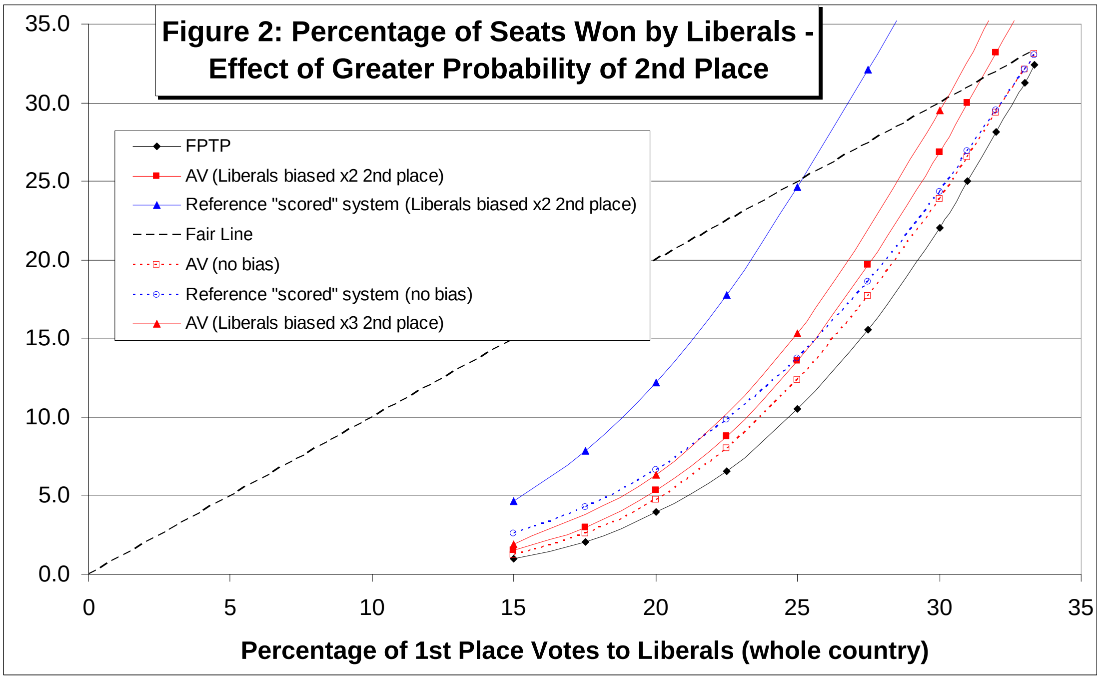


This simulation assumes that the first two parties are separated by 5% (though assuming them equal made negligible difference to Figure 1). The simulation appears to be representative in that a 26% Liberal vote would be predicted to give them ~10% of the seats, compared with the 9% which they actually won in 2010.

The main outcome of the simulation is that, whilst AV would give the Liberals more seats (perhaps 15-20 more) even AV is a very long way from proportional representation.

The blue curve in Figure 1 is the result of a system assigning a score of 2 to a 1st place vote, a score of 1 to a 2nd place vote, and zero to a 3rd place vote.

It could be argued that, based on simplistic left/centre/right considerations, the Liberals might be placed 2nd more often than 3rd (i.e., Con/Lib/Lab and Lab/Lib/Con would be more common than Lab/Con/Lib and Con/Lab/Lib). The simulation was repeated assuming that the Liberals were placed 2nd twice as frequently as 3rd. And repeated again, assuming the Liberals were placed 2nd three times as frequently as 3rd. The results are shown in Figure 2.



A 2nd place bias of x3 results in the Liberals taking a greater number of seats (perhaps ~15%, or an increase of 36 seats compare with their current number, assuming 26% of 1st place votes).

However, even in this case the number of Liberal seats falls far short of proportionality.

The simple “scored” system (blue curve) is far more sensitive and much closer to proportionality.

3. Rigorous Voting Theorems

Arrow’s Theorem (1950)

You need not waste your time searching for a fair voting system: there isn’t one when there are three or more candidates. This is the burden of Arrow’s Theorem, Ref.[1]. It is a rigorous mathematical theorem provided that a certain specific definition of “fair” is adopted. The strict mathematical statement of what constitutes fairness in this context is rather opaque. I have translated it below into language appropriate to the UK electoral system. (Apologies if this looses some rigour). Fairness requires,

[1] All voters are equal;

[2] If all voters prefer party A to party B then party A wins more seats than party B;

[3] Whether party A wins more seats than party B depends only upon the proportion of voters who prefer party A to party B (not upon the relative preference of other parties).

These seem like reasonable requirements, but actually there is no voting system which respects all three conditions (assuming three or more candidates).

Condition [1] can be weakened and the theorem remains true. It is only necessary that there is no dictator who decides the outcome unilaterally by *fiat*.

The underlying assumption is that people vote by specifying a preference order for each candidate. (FPTP is not discounted – the candidates for whom you do not vote are regarded as joint bottom place). Arrow's Theorem can be circumvented if voters are allowed to score each candidate (e.g., 80 points for your favoured candidate, 20 points for a poor second choice, and none for the hated other candidates).

Both FPTP and AV fail to meet condition [3].

Gibbard–Satterthwaite Theorem (1973)

The Gibbard–Satterthwaite Theorem, Refs.[2,3], states that providing there is no dictator and that anyone can win, there is always an opportunity for tactical voting in any electoral system. Tactical voting is defined as the ability to increase the likelihood of your desired outcome (or reduce the likelihood of your most undesirable outcome) by voting in a manner which does not reflect your true preferences. Tactical voting is possible only when the likely pattern of voting is predictable (e.g., on the basis of previous elections).

A corollary to the Gibbard–Satterthwaite Theorem is that voting systems can never be a straightforward means of joint decision making, but will always be a non-trivial game (in the strict game-theoretical sense). The first recognition of this fact has been attributed to Lewis Carroll (in his other guise as the mathematician Charles Dodgson).

Implications of the rigorous theorems for AV?

The fact that there is no system which respects all three of Arrow's requirements for fairness does not mean that all voting systems are equally unfair. It is perfectly possible for one system to be less unfair than another, and this is surely desirable. Arguably, AV ***is*** less unfair than FPTP, but both are grossly unfair with respect to proportional representation (Figures 1 and 2).

Proponents of AV have been claiming that AV eliminates the need for tactical voting. Mathematicians (e.g., David Broomhead in The Guardian, 23/4/11) have opined that this cannot be so since the Gibbard–Satterthwaite Theorem rules out this possibility. The true situation is that AV cannot entirely eliminate the opportunity for tactical voting, but nevertheless AV does make it far less common for tactical voting to be relevant. So the Electoral Reform Society and other "AV - Yes" campaigners are essentially correct despite the Gibbard–Satterthwaite Theorem.

Conclusions

[1] There is no fair voting system but some systems are less unfair than others.

[2] If the three major parties each received about one-third of the overall vote, the effect of AV on the outcome of an election could be dramatic. However this is not representative of the UK.

[3] Random simulation successfully predicts that a third party (Liberal) vote of 26% translates into ~9%-10% of the seats under FPTP.

[4] AV results in the third party (Liberals) obtaining more seats for the same percentage of the overall votes.

[5] Assuming the same total votes as in 2010 (i.e., 26%), under AV the Liberals would win 12% of the seats (75 MPs) compared with the current 9% (57 MPs).

[6] If there is also a strong tendency to place the Liberals 2^{nd}, a total of 26% of first place votes would win the Liberals 15% of seats (93 MPs) under AV.

[7] The AV system is therefore more likely to result in hung Parliaments and coalition governments.

[8] But if most people only voted for their first choice, AV would degenerate to FPTP anyway.

[9] Fairness is not the only consideration. There is also stability and decisiveness. In view of [7], FPTP may be preferable in these respects.

[10] It doesn’t matter how you vote anyway – it will always be some b****y politician in charge.

References

[1] Kenneth J. Arrow, *A Difficulty in the Concept of Social Welfare*, The Journal of Political Economy, Vol. 58, No. 4. (Aug., 1950), pp. 328-346.

[2] Allan Gibbard, *Manipulation of voting schemes: a general result*, Econometrica, Vol. 41, No. 4 (1973), pp. 587–601.

[3] Mark A. Satterthwaite, *Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions*, Journal of Economic Theory 10 (April 1975), 187–216